

Making Math Meaningful®

A 7th Grade Workbook
Teacher's Edition

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Jamie York Press

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Catherine Douglas was a student at Shining Mountain Waldorf School when she designed the cover to this book. It is an impressive example of how equiangular spirals emerge from nested octagons.



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To the Teacher

What is the intention of this workbook?

This workbook contains worksheets for seventh grade math. It is part of a developmentally based, carefully thought-out curriculum fully described in my book, *Making Math Meaningful – A Middle School Math Curriculum for Teachers and Parents*. It makes available the material I have discovered and developed while teaching middle school math for several years in a Waldorf school. I feel that this material is both exciting and age-appropriate for developing the minds of this age group. It is an alternative to the normal “they-need-all-this-stuff-in-order-to-get-ahead” approach.

Seventh grade – the beginning of abstract thinking

Seventh grade is an important year academically. This the year when students start to develop abstract thinking through science, essay writing, ratios, and an introduction to algebra. Getting homework done regularly (even in small amounts), and keeping a well-organized notebook, are both very important. It is relatively common for a student to enter seventh grade weak in math, but then to “wake-up” during seventh grade, and, in the end, to enter high school quite strong in math.

Our website

Our website is www.JamieYorkPress.com. On this site, you can view full-color photos of children’s main lesson book pages, view worked out solutions to the problems in this workbook, and download (for free!) a variety of other teaching resources. You can also contact us with questions and comments.

Required reading and resource books.

It is necessary to read our book *A Middle School Math Curriculum for Teachers and Parents* to fully understand the material in this workbook. We also recommend using our book, *Puzzles, Games and More!*, for those times when you need to do something different. This puzzle and game book is designed for fourth grade and up, and is intended to supplement our workbooks. Order our books at www.JamieYorkPress.com.

Who can use this workbook?

While most of my teaching experience is within the Waldorf school system, any teacher, who wishes to bring meaningful, age-appropriate material to their students, can use this workbook.

Warning! This is dangerous stuff!

Any workbook can be a useful tool for the students to practice a skill after the material has been appropriately and creatively presented in class – or it can be used as a bludgeon to deaden the material and torture the students.

The teacher must be familiar with all of the problems on a given worksheet before assigning it. There is a great deal to balance in the teaching of any math class: level of difficulty, amount of repetition, how much to make the students struggle, diversity of students, time needed to complete a given assignment, etc. This balancing act is the responsibility of the teacher; it is not likely to be achieved by marching through the workbook one sheet at a time.

In the end, to make a lesson come alive, the teacher should make the material his/her own and bring it creatively to the students. Using this workbook as a recipe, or teaching material that the teacher hasn’t formed a connection to, leads to uninspirational lessons and has an adverse effect on the students.

Keep it alive!

Working on skills is important, but, all too often, skills take over everything else, and the love of the subject is lost. So how do we keep the class enthusiastic about learning math as they are learning all these skills? This is the great balancing act. Here are some ideas:

- You are the author! In order to make a lesson come alive, the teacher needs to penetrate the material, make it their own, and, in the end, become the author of what is presented to the students.
- Working in groups. Rather than having the teacher at the board explaining one problem after the other, have the students explain things (e.g., homework problems that they got wrong) to each other. I also frequently give challenge problems to be worked on in groups.
- Looking ahead. It is helpful to look over the entire unit before it begins and take note of which problems are the key steps as the topic develops. Then, once things begin, I try to prepare the class for what is to come, finding ways to get them excited about the next step.
- Mix it up! Sometimes the class simply needs something different for a day or two. This can be done in a number of ways. The class could go outside for a math related activity (e.g., calculating the height of a flagpole using similar triangles). An entire lesson, or a portion of a lesson, could be spent doing a math puzzle (usually in groups), or playing a math game. Our puzzle and game book is a great resource for this.

Homework and Pace

- Getting through the material. How much of this workbook a teacher can cover during the year will depend upon the teacher, the class, and the amount of time available. This workbook assumes a well-prepared class coming into seventh grade. Obviously, it is important not to rush through the material for the sake of “getting through the curriculum.” It is better to cover less material, but do it thoroughly.
- Homework frequency. What appears in this workbook is literally the entire year of homework for my seventh grade classes. Usually, I give my seventh graders one or two homework assignments per week – each one consisting of a sheet (two pages) from this workbook.
- Different needs. There is a range of abilities in any class. Yet, I believe strongly that math classes should not be divided into slower and faster-paced classes until ninth grade. Some students can finish the assignments in this workbook relatively quickly; for other students, the assignments are quite challenging. For those few who can’t quite manage the same workload as the rest of the class, I meet with their parents and form an ILP (individualized learning plan) in order to best meet their needs. At times, this includes reducing the number of problems that are required for homework.
- Carefully choose your assignments. Much of the material found in this workbook is quite challenging. The teacher’s job is a balancing act – challenging the students, and yet making sure they are not overwhelmed. Therefore, the teacher should carefully read through each sheet before assigning it. It may be that an entire sheet is too much as a single homework assignment. The teacher can, of course, assign selected problems from any given worksheet, or leave certain problems as optional challenge problems.
- “Green” and “Red” Problems. I strongly recommend that the teacher carefully look over the homework sheet and decide which problems are “red” problems (the challenging problems that only certain students are required to do) and which are the “green” problems (the more basic problems that aren’t required for those students doing the “red” problems). A given teacher may have a different, more effective system, but the idea is that assigning all of the problems to all of the students may be ineffective or counterproductive.
- Corrections. I do not collect and correct homework assignments. My three main reasons for this are: (1) personal sanity; (2) students should feel it is OK to make mistakes on homework; (3) students need to become responsible for their own corrections. I read out the homework answers at the start of class (on the day the assignment is due), and then go over selected problems.

The Nine Units in this workbook are:

1. Arithmetic. This brief unit mostly reviews sixth grade, and includes fractions, decimals, long division, divisibility, and powers and roots. This unit continues into the review section of the measurement unit.
2. Measurement. This unit covers both the US system (hopefully, mostly a review) and the metric system.
3. Ratios – Part I. This is one of the most difficult and important units in the workbook. It gets to the essence of what a ratio really is.
4. Percents. This builds upon the percents that were introduced in sixth grade.
5. Ratios – Part II. This unit builds upon the first unit on ratios. It introduces the idea of irrational numbers, including π .
6. Rates. This includes many rate word problems, such as rate of speed and rate of pay.
7. Geometry. This unit builds upon topics introduced in the geometry main lesson, such as: angle theorems, the Pythagorean theorem, and area (the *Shear and Stretch*).
8. The Square Root Algorithm (optional). This unit consists of one of the most difficult and most rewarding topics that I have ever taught at any grade level. It should only be attempted with classes that are well prepared, and able to deal with struggling. (I only attempt it with about half of my seventh grade classes.) The teacher needs to be sure that they have completely read through and understand the entire unit before beginning it with the students. Once begun, the whole unit should be completed.
9. Algebra. This unit appears as the last unit, but is intended to be done in the middle of the year as part of the algebra main lesson. (See *Algebra Main Lesson* below.)

The ideas behind this workbook.

- Expectations from sixth grade. This seventh grade workbook is a continuation of material introduced in my sixth grade workbook. If the class didn't work with my sixth grade workbook, then it may be necessary to begin the seventh grade year with a few weeks of getting them "up to speed".
- Built-in review. I have carefully planned out how one worksheet flows into the next, and how each topic gets adequate review during the course of the year. Within a given unit, the concepts are gradually built up from one sheet to the next. Each sheet ends with a section that reviews previously covered units.
- Ratios. If there is a central theme to my seventh grade math curriculum, it is ratios. I feel that ratios are sadly neglected in today's schools. At best, ratios are incorporated into an algebra course where students are shown how to solve ratio problems in a very mechanical way (e.g., "*this is to that as this is to that*") without really developing a deep understanding of them. What I attempt to do is quite radical – guiding students to an understanding of the essence of a ratio. In the end, they should be able to solve ratio problems without using the crutch of algebra. I have integrated some original methods for learning ratios into this workbook (e.g., the "three thoughts" associated with a ratio, and the "four ratios" of a circle). This will be foreign to any teacher, but I urge you to persevere – the end result is well worth the effort!
- Classroom hours. At my school, the seventh grade math *track class* meets three times per week for 40 minutes throughout the year (a total of about 65 classroom hours). Additionally, seventh grade usually has two math main lessons (*geometry* and *algebra*), each one consisting of 1½ to 2 hours every day for about three weeks.
- The Algebra main lesson. (See *A Middle School Math Curriculum* for an explanation of the Waldorf main lesson system.) It is assumed that algebra is introduced in a three-week main lesson block. Many ideas for material to cover during this main lesson can be found in *A Middle School Math Curriculum*. *The sheets from the algebra unit in this workbook are designed to be given in the second and third weeks of the main lesson*, after formulas have been covered in a fair bit of detail, and negative numbers have been introduced. The sheets may be largely done as group work. The intention is to progress slowly. The problems remain quite simple to ensure that all of the students become solid in this important foundation of algebra. If time is short with the year-long main lesson rotation, then it is possible (with careful planning) to combine algebra and geometry into one three-week main lesson. I have done this several times quite successfully.
- The Geometry main lesson. The worksheets found in the geometry unit of this workbook are not intended for use during the geometry main lesson. Some topics (e.g., the *Golden Mean*, geometric drawing, triangle constructions) should be covered exclusively in the geometry main lesson. Other topics (e.g. angle theorems, the Pythagorean theorem, area using the *Shear and Stretch*) are introduced in the main lesson and then furthered in the track classes by using the geometry unit in this workbook. Many ideas for material to cover during this main lesson can be found in *A Middle School Math Curriculum*.
- Timing issues. The second ratios unit deals with direct and inverse proportions, so the physics main lesson (which usually covers the *Law of the Lever* and *Acoustics*) should be scheduled towards the end of the year. Also, the geometry main lesson should be scheduled before this workbook's geometry unit is covered.

Mental Math.

I strongly believe it is important for students to develop the ability to do calculations in their head. Each worksheet therefore has a section on mental math, which is based upon the math tricks found at the back of this workbook. In seventh grade, I also begin nearly every lesson with mental math. The very first sheet in this workbook assumes a familiarity with my sixth grade math tricks, and assumes that the students have memorized (or are working on memorizing) facts from each of the following areas (see *A Middle School Math Curriculum*):

1. *Multiplication facts.* Including selected facts from the 13, 14, 15, 16, 18, 25's table (e.g., $15 \cdot 5 = 75$).
2. *Powers with base 2 through 5* (e.g., $3^4 = 81$).
3. *Fraction to decimal conversions* (e.g., $\frac{3}{8} = 0.375$).

Special thanks to John Grant McLoughlin for his editing and many helpful suggestions, and to Steven Wyckoff for his help with the measurements unit.

To the Student (and Parent)

Seventh grade is perhaps the most important year academically. In math, seventh grade is when “real math” begins; up until now, you have been learning mostly just arithmetic. Some of the key themes for this year are measurement, ratios, percents, the Pythagorean Theorem, and algebra. All of this will be needed for studying math in eighth grade, and especially in high school. Remember that struggling can be an important part of learning math. Often, students enter seventh grade feeling weak in math, but with hard work and perseverance, they leave seventh grade strong in math and ready to study many interesting and challenging topics in the coming years.

Here are some tips on how to use this workbook successfully:

- Make sure your work is readable and easy to follow.
- If there isn't enough room on the worksheet, then show your work on a separate sheet, making sure you write down the worksheet number and problem number, so you can easily find it later.
- If you get stuck on one problem, then go on to another problem, and come back later to the one that you were stuck on.
- While working on a problem from one worksheet, it may be helpful to refer back to a problem that you did on a previous worksheet.
- *Fractions.* All answers to fraction problems should be reduced. Don't give answers as improper fractions, but, instead, convert them to mixed numbers (for example, leave your final answer as $3\frac{1}{4}$ instead of $\frac{13}{4}$).
- *Division.* Answers for division problems may be rounded to three significant digits, unless the problem states you should leave your answer as an exact decimal, in which case you must continue until it repeats or ends. For example, $2579 \div 56$ has an exact answer of 46.053571428 . Rounding it to three significant digits means that we go only as far as the fourth digit (which is the second place after the decimal point, and is a 5 in this case), and then round up the previous digit for an answer of 46.1.
- *Answers involving time.* Answers requiring a measure of time should be given in separated units. Examples of this are: 1 day 6 hours instead of 1.25 days and 3 hours 12 minutes instead of 3.2 hours.
- *Math Tricks!* A list of the math tricks can be found at the back of this workbook.
- Above all, homework is for learning! Try your best on every problem. Struggling and overcoming frustration are part of the process of doing math. Even if you don't get a problem correct, you will learn by trying it, and then later seeing how it should be done. Do not fall into the trap of doing the homework just to get it done.
- *Learn from your mistakes!* When you get a problem wrong, make sure you follow up on it; find your mistake, and learn how to do the problem correctly.

Getting Help. The problems in this workbook are based upon the material found in our curriculum book, titled: *A Middle School Math Curriculum for Teachers and Parents*, which can be purchased at www.JamieYorkPress.com. The book has helpful explanations and examples, and is useful for parents (or tutors) who are helping their children with the worksheets in this workbook.

Arithmetic – Sheet #1

Do it in your head

- 1) $400 \cdot 3000$
- 2) $8.46 \div 100$
- 3) $8.46 \cdot 1000$
- 4) $49 \cdot 11$
- 5) $42000 \div 600$
- 6) $3.5 \cdot 4$
- 7) $105 \cdot 108$
- 8) $512 - 497$
- 9) $3 \cdot 999$
- 10) $24 \cdot 99$
- 11) $3.6 \cdot 5$
- 12) $3.6 \div 5$
- 13) $27 - 3.7$
- 14) $0.3 \cdot 0.008$
- 15) $0.4 \div 0.008$
- 16) 13^2
- 17) $25 \cdot 6$
- 18) 3^4
- 19) 5^3
- 20) What is half of $\frac{8}{13}$?
- 21) What is half of $\frac{7}{13}$?

Quickly Estimate.

- 22) $485,036 + 225,672$
- 23) $7364 \cdot 587$
- 24) $55,963 - 42,027$
- 25) $5273 \div 886$

Division. Leave your answers as exact decimals (perhaps repeating). Use short division for single digit divisors.

26) $25,286 \div 47$

27) $4277 \div 25$

28) $0.0073 \div 0.06$

29) $7809 \div 1.37$

Fractions & Decimals

30) Convert fractions to decimals and decimals to fractions.

a) $\frac{93}{100}$

b) $\frac{9}{1000}$

c) $\frac{3}{5}$

d) $\frac{5}{6}$

e) $\frac{8}{11}$

f) $\frac{7}{24}$

g) 0.07

h) 0.043

i) 0.55

j) $0.\overline{3}$

k) 0.875

31) Convert to a mixed number.

$$\frac{45}{7}$$

32) Convert to an improper fraction.

$$6\frac{4}{9}$$

33) $\frac{5}{6} + \frac{2}{5}$

34) $\frac{48}{49} \cdot \frac{35}{48}$

35) $5\frac{3}{5} \cdot 1\frac{3}{7}$

36) $5\frac{3}{5} - 1\frac{3}{7}$

37) $5\frac{3}{5} \div 1\frac{3}{7}$

38) $\frac{5\frac{3}{5}}{1\frac{3}{7}}$

39) $(2\frac{1}{3})^2$

40) $48.3 + 1.24$

41) $48.3 - 1.24$

42) $48.3 \cdot 1.24$

Powers & Roots

43) $(8)^2$

44) $(800)^2$

45) $(0.8)^2$

46) $(0.008)^2$

47) $(12)^3$

48) $(0.1)^5$

49) $\sqrt{64}$

50) $\sqrt{9000000}$

Arithmetic – Sheet #2

Do it in your head

- 1) $5.723 \cdot 100$
- 2) $435.7 \div 100$
- 3) $2.6 \cdot 11$
- 4) $0.14 \div 4$
- 5) $21 \div 33$
- 6) 15^2
- 7) $25 \cdot 5$
- 8) 25^2
- 9) 4^3
- 10) 5^4
- 11) $700 \cdot 80$
- 12) $160,000 \div 800$
- 13) What is $\frac{9}{20}$ doubled?
- 14) What is $\frac{9}{19}$ doubled?
- 15) $8.5 \cdot 4$
- 16) $1110 \cdot 1080$
- 17) $6023 - 5996$
- 18) $9999 \cdot 4$
- 19) $999 \cdot 14$
- 20) $6400 \cdot 5$

Divisibility. State whether each number is evenly divisible by anything from 2 to 12 (but not 7).

21) 1,033,857

22) 1,378,416

Division. Leave your answers as mixed numbers. Use short division for single digit divisors.

23) $1033857 \div 11$

24) $197400 \div 389$

25) $1378416 \div 9$

Powers & Roots

26) $(600)^2$

27) $(5.42)^2$

28) $(10)^5$

29) $(1)^{31}$

30) $(0.052)^3$

31) $(\frac{3}{4})^2$

32) $(\frac{3}{4})^3$

33) $\sqrt{4900}$

34) $\sqrt{1000000}$

35) $\sqrt[3]{1000000}$

36) $\sqrt[6]{1000000}$

Fractions & Decimals

37) Convert to a fraction.

- a) 0.003
- b) 0.08
- c) 0.0125
- d) $0.\overline{5}$
- e) $0.\overline{6}$

38) Convert to a decimal.

- a) $\frac{4}{5}$
- b) $\frac{2}{11}$
- c) $\frac{3}{20}$
- d) $\frac{13}{99}$
- e) $\frac{11}{25}$
- f) $\frac{19}{60}$

39) Convert to a mixed number.

$$\frac{67}{12}$$

40) Convert to an improper fraction.

$$10\frac{3}{7}$$

41) Reduce.

a) $\frac{210}{490}$

b) $\frac{12600}{27000}$

c) $\frac{27000}{43875}$

42) $\frac{5}{6} - \frac{1}{4}$

43) $\frac{5}{9} + \frac{21}{25}$

44) $\frac{5}{9} \cdot \frac{21}{25}$

45) $78\frac{2}{3} - 76\frac{3}{4}$

46) $\frac{3\frac{3}{4}}{5}$

47) $33 \div 3\frac{2}{3}$

Quickly Estimate.

48) $693 \cdot 79$

49) $2317 - 1824$

50) $51,893 + 16,256$

51) $36478 \div 491$