---- Trigonometry -- Part II ----

7) The ratio of the lengths of the sides of a golden rectangle is Φ :1, where $\Phi = \frac{\sqrt{5}+1}{2}$.

Calculate the angle formed by the diagonal and the shortest side.

8) There are two fire towers where one is 12 miles to the north of the other. A fire is spotted in the forest that is S43°E (43° east of south) from the north tower, and N52°E from the south tower. What is the distance from the fire to the closer fire tower?

Problem Set #6

1) Find the variable indicated.



2) Find all the missing sides and angles. $10^{-22^{2}}$

Word Problems

- 3) What is the length of the shadow of a 25-foot tall pole if the angle of elevation of the sun is 30°? (Assume that the ground is flat.)
- 4) How tall is a pole with a 35-foot shadow if the angle of elevation of the sun is 25°?

- 5) A 18-foot tall pole casts a 15¹/₂-foot shadow. What is the angle of elevation of the sun?
- 6) Calculate the length of the longest diagonal of a regular heptagon (7-gon) that has sides of length 1.
- 7) The Great Pyramid of Giza was built with a base length of 756 feet and an overall height of 481 feet.
 - a) What is the angle of inclination of the triangular faces?
 - b) What is the length of the edge coming down from the peak of the pyramid?
 - c) What are the base angles of the triangular faces of the pyramid? (Where else have you seen this angle?)
- 8) The Tower of Pisa now stands about 4.0° off vertical. If the sun is opposite the direction of the leaning and at a 55° angle of elevation, then the shadow (measured from the side of the tower) would be about 43.0m long. How high above the ground is the low side of the tower? (Assume the tower is a cylinder.)

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- Cartesian Geometry - Part II -

Problem Set #2

Graphing Parabolas

In our previous on *Cartesian Geometry* we learned a method for easily graphing lines. At this point, if the equation doesn't produce a linear graph, we have make a table and then plot points.

We will now learn an easier method for graphing parabolas.

- 1) How can we tell by quickly looking at an equation whether it is a parabola?
- 2) Look at the below graph of the equation $y = x^2$. Answer the following questions:



- a) How would the graph of $y = x^2 + 3$ be different than $y = x^2$? Graph it.
- b) How would the graph of $y = x^2 3$ be different than $y = x^2$? Graph it.
- c) How would the graph of $y = (x+3)^2$ be different than $y = x^2$? Graph it.
- d) How would the graph of $y = (x-3)^2$ be different than $y = x^2$? Graph it.

- e) How would the graph of $y = -x^2$ be different than $y = x^2$? Graph it.
- f) How would the graph of $y = 2x^2$ be different than $y = x^2$? Graph it.
- g) How would the graph of $y = (x+3)^2 + 2$ be different than $y = x^2$? Graph it.
- h) How would the graph of $y = (x-4)^2 - 6$ be different than $y = x^2$? Graph it.
- i) How would the graph of $y = 2(x+1)^2-3$ be different than $y = x^2$? Graph it.
- 3) Graph each of the
 - following:
 - a) y = 2x 4
 - b) $y = \frac{x}{3} + 1$
 - c) 6x 5y = 10
- 4) Give the equation of the line that...
 - a) Passes through the points (3,2) and (-1,1).
 - b) Passes through the point (-4,-2) and runs parallel to the line 2y 3x = 12
- 5) Jack needs to rent a car for one day. Happy Rent-a-Car is offering a special on economy cars for \$25 for the day, plus 3 e/mile. The best deal at Ken's Car Rental is for \$15/day plus 7¢/mile. For each of the two companies, give a function of the total cost with respect to miles driven, then graph these functions (up to 500 miles driven). Under what circumstances should Jack choose each of the two rental companies?

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— Cartesian Geometry – Part II —

Using a Harmonic Net

Another coordinate system that is even less well known, but is still very interesting, uses a *harmonic net*, a form from projective geometry.

The below drawing can be imagined as a checkerboard in two-point parallel perspective. Can you figure out how to draw it?

If we draw the diagonals of the "squares", then we get two more points on the same line as the two vanishing points. These two new points are harmonic conjugates of the two original vanishing points.

3) With the below drawing, choose one of the lower "horizontal" lines as the x-axis, and one of the center "vertical" lines as the y-axis. Plot points for $y = x^2$. Can you picture where the point would be for an infinitely large x? What is the resulting curve?

- 4) Using the curve from the previous problem, imagine "nailing down" two points (e.g., (1,1) and (-2,4)) and then sliding the top vanishing point infinitely far upwards, and the right vanishing point infinitely far to the right. This makes it so that the horizontal lines are evenly spaced, and the vertical lines are evenly spaced. What does the curve look like now?
- 5) Considering the drawing from problem #3, what would need to be changed in order that the resulting curve would be a hyperbola?

For the rest of this unit, we will graph equations using the standard Cartesian coordinate system. But keep in mind that any equation can instead be graphed on a different coordinate system, resulting in a completely different curve.



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- Possibility & Probability- Part II -

Problem Set #3

- 1) How many ways can these letters be rearranged:
 - a) RUNNER?
 - b) ERROR?
- c) AAABBCCCCD?
- 2) In how many different ways can a 7-question multiple-choice test be answered if every question has A, B, C, or D as its options?
- 3) Fred needs to visit four cities. How many possible ways are there for the order in which to visit the cities?
- 4) How many ways can you choose 59 things out of 60 (without regard to order)?
- 5) An outing club has a membership of 4 women and 6 men. A social committee of 4 is to be formed. In how many ways can this be done if...
- a) there must be 2 women and 2 men on the committee?
- b) there must be at least 1 woman on the committee?
- c) all 4 must be of the same sex?
- 6) A railroad line has 8 stops. How many different one-way tickets are possible?
- 7) How many different ways are there to arrange 8 identical large chairs and 3 identical small chairs in a row?
- 8) What is the probability of randomly, but correctly, guessing the top three finishers in a 20-person race?

- 9) Two dice are rolled. Find the probability that...
- a) the sum of the numbers showing on the dice is 5.
- b) the sum of the numbers showing on the dice is 11.
- c) you get a 6 on exactly one die.
- d) you get a 6 on at least one die.
- 10) A bag has 2 red, 4 pink, and 6 blue marbles in it. Two marbles are drawn at random. Find the probability that...
- a) both are red.
- b) one is red and one is pink.
- c) neither is red.
- 11) Five coins are tossed. Find the probability of ...
- a) getting all heads?
- b) getting one head?
- c) getting two heads?
- d) getting at least three heads?
- 12) What is the probability of drawing a red card or a 5 from a standard 52-card deck?
- 13) Ten identical coins are to be distributed randomly between four people. How many ways can this be done?
- 14) With a 13-card hand, what is the probability of getting...
- a) only red cards?

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- b) no face cards (J, Q, K)?
- c) at least one face card?
- d) one card of each kind (one ace, one king, etc.)?
- e) exactly 11 diamonds?

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