$\sqrt{}$ Algorithm − Sheet #6

1) Calculate each square root using the *Long Algebraic Method*, as done on the previous worksheets. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$

The Short Algebraic Method

<u>The Basic Idea</u>

- *Reducing the amount of Calculating*. The long algebraic method, described above, requires some tedious, and unnecessary, calculations, which can be eliminated.
- Look at example for the long algebraic method shown on sheet #4. Looking at the left side of each step, we see, for step #1: $n - a_1^2$, and then for step #2: $n - a_2^2$, etc.
- Since $a_2 = a_1 + b_1$, we can use the *Squaring Formula* $(a+b)^2 = a^2 + b(2a+b)$ to get:

$$a_2^2 = (a_1 + b_1)^2 = a_1^2 + b_1(2a_1 + b_1)$$

<u>This is the key idea</u>: In place of subtracting a_2^2 from n, we can instead subtract the whole of $\{a_1^2 + b_1(2a_1 + b_1)\}$ from n since it is equal to a_2^2. This seems like more work, but it's not - it's less work.

In other words, instead of doing $n - a_2^2$, we can do $n - \{a_1^2 + b_1(2a_1 + b_1)\}$,

which is the same as $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$

In short: instead of doing $n - a_2^2$ we do $(n - a_1^2) - \{b_1(2a_1 + b_1)\}$

Likewise, instead of doing $n - a_3^2$ we do $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$

Likewise, instead of doing $n - a_4^2$ we do $(n - a_3^2) - \{b_3(2a_3 + b_3)\}$, etc.

Of course, any sane person would ask, "Haven't we made things more complicated?". The answer to this is (and this is where the genius of this method comes in): $(n - a_2^2) - \{b_2(2a_2 + b_2)\}$ is easier to do than $n - a_3^2$ because a_3^2 requires us to square some big ugly number (e.g. 2680), whereas we have already calculated both $(n - a_2^2)$ (which is 443856 in the example below) and $\{b_2(2a_2 + b_2)\}$ (which is 422400 in the example below).

Subtracting 443856 − 422400, is easier than squaring 2680!!!!!

Example: $\sqrt{7203856}$ (once again!):

	n	7203856	our first estimate (a_1) is 2000.
	$\mathbf{a_1^2}$	− 4000000	
<u>step #1</u>	$\mathbf{n - a_1^2}$	3203856	$= b_1(4000 + b_1) \rightarrow$ <u>$b_1 = 600$</u>
	$\mathbf{b_1(2a_1 + b_1)}$	− 2760000	
<u>step #2</u>	$\mathbf{n - a_2^2}$	443856	$= b_2(5200 + b_2) \rightarrow$ <u>$b_2 = 80$</u>
	$\mathbf{b_2(2a_2 + b_2)}$	− 422400	
<u>step #3</u>	$\mathbf{n - a_3^2}$	21456	$= b_3(5360 + b_3) \rightarrow$ <u>$b_3 = 4$</u>
	$\mathbf{b_3(2a_3 + b_3)}$	− 21456	
		0	So our answer is *exactly* <u>**2684**</u>

2) Calculate each square root using the *Short Algebraic Method*. It is important that you do the problem and organize your work exactly like the example given above. Notice that the first three problems are the same ones given in the previous exercise. (All answers work out exactly.)

a) $\sqrt{403225}$ b) $\sqrt{61009}$ c) $\sqrt{24137569}$ d) $\sqrt{393129}$ e) $\sqrt{145924}$