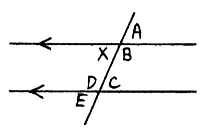
Geometry Basics

Problem Set #1

Group Work

Note: In this unit you need a compass, protractor, and ruler.

Look at the drawing below. With #1 through #5, fill in the sentence with the proper words given the choices below.



- vertical angles
- supplementary angles
- corresponding angles
- alternate interior angles
- same-side interior angles

1) X and B are ________.
2) X and C are ________.
3) X and A are ________.
4) X and D are ________.
5) X and E are ________.

Angle Theorems

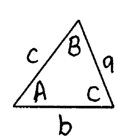
Complete the sentence either with "are congruent" or "add to 180°."

6) *Vertical Angle Theorem.* Vertical angles always…
7) *Supplementary Angle Theorem.* Supplementary angles always…
8) *Corresponding Angle Theorem.* Corresponding angles always…
9) *Alternate Interior Angle Theorem.* Alternate interior angles always…
10) *Same-Side Interior Angle Theorem.* Same-side interior angles always…
11) *Triangle Interior Angle Theorem.* The angles in a triangle always…

Triangle Constructions (by measuring)

With the below table, each row represents a triangle, with three given measurements. Use a ruler and protractor (and perhaps a compass) to construct each triangle on a clean sheet of paper. Then measure (accurately!) to fill in the rest of the table.

	a	b	c	A	B	C
12)	5cm	7cm	6cm			
13)	7cm	7cm	3cm			
14)	6cm	4cm				110°
15)			7cm	40°	20°	
16)			7cm	40°		120°
17)	7cm			40°	20°	



Circle Geometry

Group Worksheet

Note: The exercises on this group worksheet are intended to allow you to discover the theorems of *Circle Geometry*. It may be best to save some of these group exercises until later, after some of the problem sets have been worked through.

1) *Parallel Chord Th.*
   Draw five circles and with each one draw a pair of parallel chords. Do each one differently so that in some cases the chords are of various distances apart, of various lengths, sometimes both on the same side of the circle's center, and sometimes on opposite sides. State the *Parallel Chord Theorem* (i.e., What is always true if there are two parallel chords in a circle?)

2) *Equal Chord Theorem.*
   What can be said about any two equal-length chords within a circle regarding the arcs that they subtend?
   How can we be sure that this is always true?

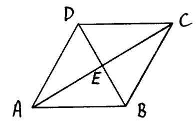
3) *Intersecting Chord Th.*
   Draw a circle with a radius of about 6cm, and mark its center. Draw any two chords inside the circle so that they intersect. Label the acute angle formed by the intersecting chords as $\angle C$. Now label the arc subtended by $\angle C$ as arc A, and the opposite arc as arc B. Use a protractor to measure the arcs A and B and $\angle C$.
   Now repeat the same process with four more circles, but by having each pair of chords intersect very differently. Record your results in a table. Give a formula that relates A, B and C. How can we be sure that this is always true?

4) *Inscribed Angle Th.*
   Draw a circle and an angle that is inscribed within that circle (i.e., the vertex of the angle sits on the circle). Measure the angle and the arc that it subtends.
   a) Give a formula relating the measure of the angle and the arc.
   b) How can we be sure that this is always true?
   c) If the arc is 180°, then what can be said?
   d) What is the name of the theorem that makes a statement about the above special case?

5) *Chord-Tangent Theorem*
   Draw a circle and a tangent to that circle. From this point of tangency draw a chord. What can be said about the relationship of the angle formed by the intersection of the chord and the tangent, and the arc inside this angle?
   How can we be sure that this is always true?

15) <u>Rhombus Diagonal Theorem</u>.

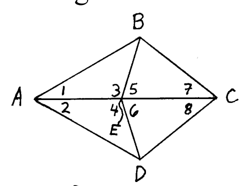
a) What property can be stated regarding the diagonals of any rhombus? Prove it.



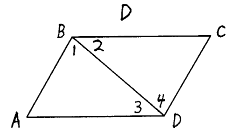
b) Given rhombus ABCD, and AB = BD = 8, find the area of the rhombus, and the area of the rectangle formed by BD and AC?

c) For any rhombus, what is the ratio of the area of the rhombus to the area of the rectangle formed by the diagonals?

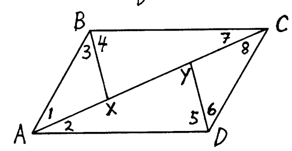
16) <u>Given</u>: $AB \cong AD$; $BC \cong CD$.
<u>Prove</u>: $BE \cong ED$.



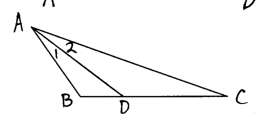
17) <u>Given</u>: $AB \parallel CD$; $AB \cong CD$.
<u>Prove</u>: $BC \parallel AD$.



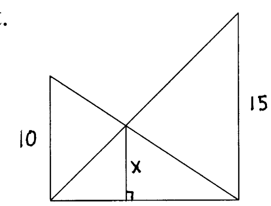
18) <u>Given</u>: ▱ABCD; $\angle 3 \cong \angle 6$.
<u>Prove</u>: $\Delta ADY \cong \Delta CBX$.



19) **<u>Triangle Angle–Bisector Theorem</u>.
(from *The Elements,* Theorem VI-3)
"An angle-bisector of a triangle divides the opposite side into segments proportional to the other two sides." Prove it.



20) ** With the diagram shown here, the lines marked as 10, 15, and x are all parallel. It is interesting to note that if the base of the figure were lengthened or shortened, the value of x would not change. Find x.



21) **<u>Eratosthenes' Measurement of the Earth</u>

Around 230 B.C., Eratosthenes calculated the circumference of the earth by using only two measurements. First, at summer solstice, when the sun could shine to the bottom of a well in Syene (which is on the Tropic of Cancer), he measured that the sun at Alexandria (nearly directly north of Syene) was 7.2° off from being directly overhead. Secondly, he calculated, based upon the time it took a camel train to go from Alexandria to Syene, that the distance between the two cities was 5000 stadia.

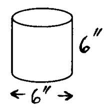
a) What was Eratosthenes' value for the earth's circumference?

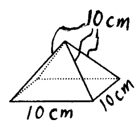
b) Given that the actual (average) radius of the earth is about 6371km, and that 1 stadium is believed to be about 157m (and this is disputed!), what was Eratosthenes percent error?

Problem Set #4

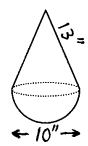
Section A

1) Find the volume and surface area.

a) 

b) 

2) Find the volume.



3) Find the volume of an octahedron with 3cm edges.

4) Find the diameter of a sphere with a volume of 30 ft^3.

5) Find the diameter of a sphere with a surface area of 30 ft^2.

6) If the ratio of the radii of two circles is a:b, then what is the ratio of their areas?

7) Fill in the blanks.

a) If the scale factor of two similar figures (e.g., two circles, or two rectangles) is a:b, then find the ratio of their areas.

b) If the scale factor of two similar solids (e.g., two cylinders, or two prisms) is a:b, then find the ratio of their volumes.

8) If one log is 60% longer and 60% thicker than another log, how much does the bigger one weigh if the smaller one weighs 10 pounds?

9) A rectangle has an area of 15 and a perimeter of 17.

a) Find the dimensions of the rectangle.

b) Find the perimeter of a square with the same area.

c) Find the circumference of a circle with the same area.

10) If the base of a cone has an area of $20m^2$, what is the area of the section parallel to base that is halfway up the cone?

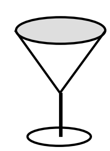
Section B

11) Derive a formula for the area of an equilateral triangle given B as its base.

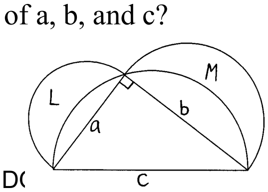
12) Find the surface area of a cone (disregarding its base) that has a base radius of 8m and an edge length of 10m.

13) Derive a formula that calculates the surface area of a cone (disregarding the circular base) given k as the length along the edge, and r as the radius of the base.

14) *The Conical Drinking Glass.* How much water is in a conical drinking glass that is filled half-way to the top, if its maximum capacity is 12 fl. oz.?



15) *Leonardo da Vinci's Lunes.* What is the combined area of the two lunes (L & M) in terms of a, b, and c?



Problem Set #7

Simplify.

1) $(2x^3y^2)^2(-2xy^3)^3$

2) $(x+2)(x-2)^2$

3) $\dfrac{x^2-10x+21}{x^2+10x-39}$

4) $\dfrac{3}{4x^2y}+\dfrac{5}{6xy^3}$

5) $\dfrac{\dfrac{\frac{25}{x}-x}{x}-3}{2x^2-5x}$

Factor.

6) $x^2+25x-70$

7) $5x^2+25x-70$

8) x^2-100

9) x^6-10

10) x^4+25

11) $9x^4-25x^6$

12) $5x^2+23x-10$

Solve for X in terms of Y.

13) $Y=\frac{2}{3}X-6$

Find the Common Solution.

14) $x+2y=1$
$3x-4y=23$

Solve.

15) $\frac{x-2}{2x-25}=\frac{3}{x+20}$

16) $5x+2=-3x^2$

17) $(x-3)^2=3x^2+4x+12$

18) $\frac{4x}{x-2}=\frac{x-5}{x-3}$

19) $-\frac{3}{5}X-4=\frac{5}{6}-X$

20) $(2x+3)(2x-3)=-x-6$

21) $\frac{x}{x+1}+\frac{1}{x-2}=\frac{3}{x^2-x-2}$

22) $\frac{2}{x-1}+\frac{8}{1-x^2}=\frac{3}{x+1}$

23) $10x=3x^2+8$

24) $100x^2+5x^4=6x^4$

25) Find the area of the isosceles right triangle triangle that has a perimeter of 10. (leave answer in radical form.)

26) Bill jogged around a track averaging 3 m/s on the first lap and then 4 m/s on the second lap. The combined time for the two laps was 3 minutes 30 seconds. What is the perimeter of the track?

Problem Set #3

Deriving the Laws of Logarithms!

Calculate each. Use the *Power and Base Tables*, as needed.

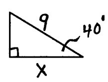
1) a) $\log_2 16$
   b) $\log_2 64$
   c) $\log_2 (64\cdot 16)$
2) a) $\log_{10} 1000$
   b) $\log_{10} 100{,}000$
   c) $\log_{10} (100{,}000\cdot 1000)$
3) $\log_3 (9\cdot 27)$
4) Derive a Law of Logarithms!
   $\log_b (M\cdot N) =$
5) a) $\log_{10} 100{,}000$
   b) $\log_{10} 1000$
   c) $\log_{10} (100{,}000\div 1000)$
6) a) $\log_3 2187$
   b) $\log_3 243$
   c) $\log_3 (2187\div 243)$
7) $\log_2 (512\div 32)$
8) Derive a Law of Logarithms!
   $\log_b (M\div N) =$
9) a) $\log_2 8$
   b) $\log_2 (8^3)$
10) a) $\log_{10} 1000$
    b) $\log_{10} (1000^5)$
11) $\log_3 (9^7)$
12) Derive a Law of Logarithms
    $\log_b (N^k) =$
13) a) $\log_2 8$
    b) $\log_2 (^1/_8)$
14) a) $\log_{10} 100{,}000$
    b) $\log_{10} \left(\frac{1}{100000}\right)$
15) How are $\log_b (^1/_N)$ and $\log_b N$ related to one another? Write a Law of Logarithms that expresses this.
16) a) $\log_3 81$
    b) $\log_{81} 3$
17) a) $\log_{10} 100$
    b) $\log_{100} 10$
18) How are $\log_a b$ and $\log_b a$ related to one another? Write a Law of Logarithms that expresses this.
19) $\log_3 (3^7)$
20) $\log_{10} (10^6)$
21) Derive a Law of Logarithms
    $\log_b (b^k) =$
22) $5^{\log_5 625}$
23) $10^{\log_{10} 1000}$
24) Derive a Law of Logarithms
    $b^{\log_b N} =$
25) What can the following Logarithm Law be used for?

$$\log_a x = \frac{\log_b x}{\log_b a}$$

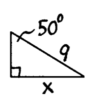
Problem Set #5

A new Trigonometric Function: *Cosine*

As you can see from the last two problems on the previous problem set, finding the length of the leg adjacent to the given angle can be more work. The easiest way to do these problems is to work with the sine of the complementary angle. (Complementary angles add to 90°.) This may seem confusing, so I'll explain.



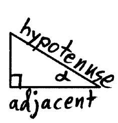
Using the example of the last problem, this means that we don't use 40°, but instead we notice that the angle at the top is 50°, and then the whole problem has been transformed into something that we can deal with – finding the length of a side that is opposite to the given angle. The triangle now looks like this:



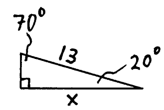
This need for working with the sine of the complementary angle is so common that it has become trigonometry's second function, the *cosine*, which means "complementary sine".

The Idea of Cosine: Given α as one of the angles in a right triangle, the value of the Cosine of the angle is abbreviated as $\cos(\alpha)$, and the following can be stated about it:

- $\cos(\alpha)$ essentially answers the question: "The adjacent leg is what proportion of the hypotenuse?"
- $\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}}$



Why is this such a big deal? Because now we have flexibility. For example, with the triangle shown below, we can either use sin(70°) or use cos(20°). Either way we get $x \approx 12.2$. (Do you understand where this answer comes from?)



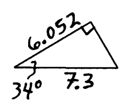
1) Give the meaning of each statement. Use the words "hypotenuse" and "adjacent leg".

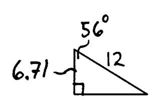
a) $\cos(41°) \approx 0.755$

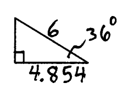
b) $\cos(83°) \approx 0.122$

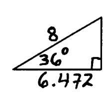
c) $\cos(18°) \approx 0.951$

2) Using the information given on each problem, and *not* using the trig buttons on your calculator, calculate the cosine of the given angle.

a) 

b) 

c) 

d) 

Problem Set #2

Intervals between Notes

Early in the quest for the hidden mathematical laws behind the beauty of music, the Pythagoreans discovered a very important property – that the most pleasing musical intervals were those that had frequencies that could be reduced to simple whole number ratios. The simpler the ratio (e.g., 3:2), the more consonant, or harmonious, the interval. Intervals with more complex ratios (e.g., 256:243) would sound more dissonant to one's ear.

We can expand further what the Pythagoreans started by creating a complete musical scale based only on simple ratios. The table below shows the basic intervals.

Table of Intervals

Name of Interval	Ratio of Interval	Example
Second	9:8	C → D
Major 3rd	5:4	C → E
Fourth	4:3	D → G
Fifth	3:2	A → E
Major 6th	5:3	C → A
Major 7th	15:8	C → B
Octave	2:1	B → B'

For each resulting note, give the name and frequency (number of vibrations per second)**.** Use the *Table of Intervals* as needed.

1) What is the frequency of the note that is a fifth up from A 440 Hz?

2) What is the frequency of the note that is a fourth up from A 440 Hz?

3) What is the frequency of the note that is a third up from F 352 Hz?

4) What is the frequency of the note that is an octave up from F 352 Hz?

5) What is the frequency of the note that is three octaves up from F 352 Hz?

6) What is the frequency of the note that is two octaves below F 352 Hz?

7) What is the frequency of the note that is 2 fifths above A 440 Hz?

8) What is the frequency of the note that is 2 fourths above A 440 Hz?

9) What is the frequency of the note that is a fifth and then a fourth above A 440 Hz?

10) What is the significance of the answer to the previous problem?