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"Regular" classes[1]

Teaching a regular math class in eleventh and twelfth grade can be quite challenging. We feel it is unfortunate if a regular math class only has an experience of working on fairly mundane math skills, and never has a chance to experience more interesting and meaningful math. Eleventh graders are not satisfied by grinding through endless problems in order to build skills – especially if they are not going to be using these skills in the future. The question therefore becomes: how can we bring interesting (and perhaps quite advanced) math topics to a group of students that may have fairly weak skills? The key to making this possible is more time, and lots of encouragement and coaching. It is appropriate for the regular eleventh grade class to receive less of an emphasis on skills than the advanced class. This group may thrive on a good dose of puzzles and problem solving. It's all about finding balance!

Prerequisites and timing

This workbook assumes that the students have studied a full year of Algebra I in ninth grade, and then in tenth grade have had a thorough review of Algebra I, as well as an introduction to trigonometry, and some basic knowledge of logarithms. All of this material is found in the second half of our *10th Grade Workbook.* However, it is <u>not</u> the case that in order to use this workbook the class needs to have covered every unit of the *10th Grade Workbook.*

Problem-solving exercises – when?

We feel it would be unfortunate if a class never got to experience any of our problem-solving exercises. These exercises do not comprise a typical unit; it is not the intention to work through them from beginning to end. This "unit" of problem-solving exercises is placed at the end of the workbook, not because it has the lowest priority, or that it should be done last. But, rather, the idea is that the teacher will frequently, over the course of the year, select a problem-solving exercise to bring to the class and allow the students the time to work through the problem. This takes both patience and careful planning on the part of the teacher. Read more about the *Problem-Solving Exercises* in the "Unit-by-Unit Commentary" below.

Sleep and review

In order for students to achieve mastery of a mathematical skill, they need to see that topic several times. The teacher needs to create a dance between introducing, deepening, practicing, sleeping, and

[1] When we refer to the "regular math class", we are assuming that the grade is split by ability and skill into two classes: the advanced class and the regular class. The regular class typically moves at a slower pace, and may have many students who are under-confident with math, and have struggled for many years with math.

—— *Unit-by-Unit Commentary* ——

The units covered in this workbook are:

1. Cartesian Geometry, Part I
2. Trigonometry, Part II
3. Complex Numbers, Part I
4. Cartesian Geometry, Part II
5. Possibility & Probability, Part II
6. Trigonometry, Part III
7. Cartesian Geometry, Part III
8. Logarithms, Part III
9. Complex Numbers, Part II
10. Trigonometry, Part IV
11. Cartesian Geometry, Part IV
12. Calculus, Part I
13. Calculus, Part II
14. Statistics, Part I
15. Statistics, Part II
16. Problem-Solving Exercises

Cartesian Geometry, Part I

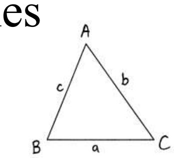
Overview

This first unit on Cartesian geometry is intended to give a strong foundation for this critically important subject at the start of eleventh grade. Indeed, in precalculus, calculus, and beyond, much of the study of mathematics assumes a strong knowledge of Cartesian geometry.

Topics covered in this unit

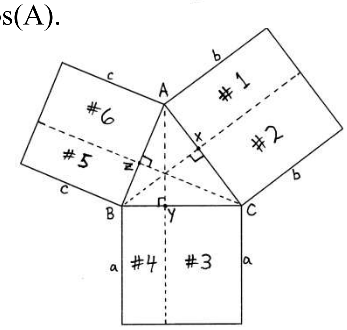
- The Cartesian coordinate axis
- The Golden Rule of Cartesian Geometry
- Graphing two-variable equations by plotting points from a table
- Overview of several different types of graphs, including lines, circles, parabolas, cubics, etc.
- Focus on graphing linear equations
- Slope of a line
- Slope-intercept form of an equation
- Standard form of an equation
- Determining equations of lines given certain conditions
- Using three methods to find common solutions to linear equations
- Word problems represented as linear equations

Proof of the Law of Cosines



1. Given acute triangle, ΔABC, attach squares to the sides of the triangle, draw the triangle's three altitudes, and label everything as shown below.
2. Using ΔABX, we see that $AX = c\cdot\cos(A)$.

   Similarly, we have
   $CX = a\cdot\cos(C)$
   $BY = c\cdot\cos(B)$
   $CY = b\cdot\cos(C)$
   $AZ = b\cdot\cos(A)$
   $BZ = a\cdot\cos(B)$



3. The areas of the six rectangles are:
   rect#3 = rect#2 = $ab\cdot\cos(C)$
   rect#5 = rect#4 = $ac\cdot\cos(B)$
   rect#6 = rect#1 = $bc\cdot\cos(A)$
4. a^2 = rect#4 + rect#3; b^2 = rect#1 + rect#2; c^2 = rect#5 + rect#6
5. rect#5 + rect#6 + rect#3 = rect#1 + rect#2 + rect#4

$$
\begin{aligned}
c^2 + \text{rect\#3} &= b^2 + \text{rect\#4} \\
c^2 + \text{rect\#3} + \text{rect\#3} &= b^2 + \text{rect\#4} + \text{rect\#3} \\
c^2 + 2(\text{rect\#3}) &= b^2 + a^2
\end{aligned}
$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cdot\cos(C)}$$

When do we use each law?

With most of these problems, we are given three pieces of information, and we are asked to find a missing side or angle – that's four things that we *care* about. (We use the word "care" quite intentionally.) The general rules for when to use each law are as follows:

- Law of Sines is used when we *care* about two sides and two angles. (The exception is when we use the Law of Tangents.)
- Law of Cosines is used when we *care* about three sides and one angle.
- Law of Tangents is used when we are *given* an SAS triangle, and want to find a missing angle. (We could also solve such a triangle by first using the Law of Cosines to find the missing side, and then using the Law of Sines to find the desired angle.)

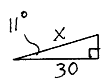
7) The ratio of the lengths of the sides of a golden rectangle is Φ:1, where $\Phi = \frac{\sqrt{5}+1}{2}$.

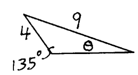
Calculate the angle formed by the diagonal and the shortest side.

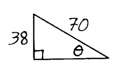
8) There are two fire towers where one is 12 miles to the north of the other. A fire is spotted in the forest that is S43°E (43° east of south) from the north tower, and N52°E from the south tower. What is the distance from the fire to the closer fire tower?

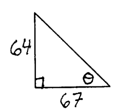
Problem Set #6

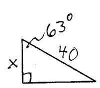
1) Find the variable indicated.

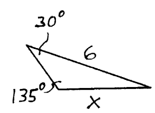
a) 

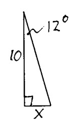
f) 

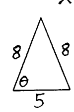
b) 

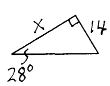
g) 

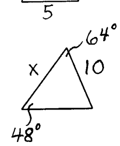
c) 

h) 

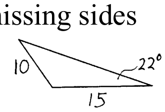
d) 

i) 

e) 

j) 

2) Find all the missing sides and angles.



Word Problems

3) What is the length of the shadow of a 25-foot tall pole if the angle of elevation of the sun is 30°? (Assume that the ground is flat.)

4) How tall is a pole with a 35-foot shadow if the angle of elevation of the sun is 25°?

5) A 18-foot tall pole casts a 15½-foot shadow. What is the angle of elevation of the sun?

6) Calculate the length of the longest diagonal of a regular heptagon (7-gon) that has sides of length 1.

7) The Great Pyramid of Giza was built with a base length of 756 feet and an overall height of 481 feet.
   a) What is the angle of inclination of the triangular faces?
   b) What is the length of the edge coming down from the peak of the pyramid?
   c) What are the base angles of the triangular faces of the pyramid? (Where else have you seen this angle?)

8) The Tower of Pisa now stands about 4.0° off vertical. If the sun is opposite the direction of the leaning and at a 55° angle of elevation, then the shadow (measured from the side of the tower) would be about 43.0m long. How high above the ground is the low side of the tower? (Assume the tower is a cylinder.)

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Problem Set #2

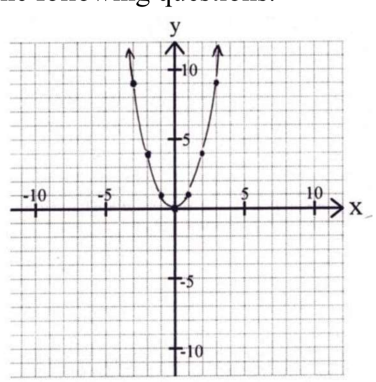
Graphing Parabolas

In our previous on *Cartesian Geometry* we learned a method for easily graphing lines. At this point, if the equation doesn't produce a linear graph, we have make a table and then plot points.

We will now learn an easier method for graphing parabolas.

1) How can we tell by quickly looking at an equation whether it is a parabola?

2) Look at the below graph of the equation $y = x^2$. Answer the following questions:



a) How would the graph of $y = x^2 + 3$ be different than $y = x^2$? Graph it.

b) How would the graph of $y = x^2 - 3$ be different than $y = x^2$? Graph it.

c) How would the graph of $y = (x+3)^2$ be different than $y = x^2$? Graph it.

d) How would the graph of $y = (x-3)^2$ be different than $y = x^2$? Graph it.

e) How would the graph of $y = -x^2$ be different than $y = x^2$? Graph it.

f) How would the graph of $y = 2x^2$ be different than $y = x^2$? Graph it.

g) How would the graph of $y = (x+3)^2 + 2$ be different than $y = x^2$? Graph it.

h) How would the graph of $y = (x-4)^2 - 6$ be different than $y = x^2$? Graph it.

i) How would the graph of $y = 2(x+1)^2-3$ be different than $y = x^2$? Graph it.

3) Graph each of the following:

a) $y = 2x - 4$

b) $y = \frac{x}{3} + 1$

c) $6x - 5y = 10$

4) Give the equation of the line that…

a) Passes through the points (3,2) and (−1,1).

b) Passes through the point (−4,−2) and runs parallel to the line $2y - 3x = 12$

5) Jack needs to rent a car for one day. Happy Rent-a-Car is offering a special on economy cars for \$25 for the day, plus 3¢/mile. The best deal at Ken's Car Rental is for \$15/day plus 7¢/mile. For each of the two companies, give a function of the total cost with respect to miles driven, then graph these functions (up to 500 miles driven). Under what circumstances should Jack choose each of the two rental companies?

Problem Set #3

1) How many ways can these letters be rearranged:
  a) RUNNER?
  b) ERROR?
  c) AAABBCCCCD?

2) In how many different ways can a 7-question multiple-choice test be answered if every question has A, B, C, or D as its options?

3) Fred needs to visit four cities. How many possible ways are there for the order in which to visit the cities?

4) How many ways can you choose 59 things out of 60 (without regard to order)?

5) An outing club has a membership of 4 women and 6 men. A social committee of 4 is to be formed. In how many ways can this be done if…
  a) there must be 2 women and 2 men on the committee?
  b) there must be at least 1 woman on the committee?
  c) all 4 must be of the same sex?

6) A railroad line has 8 stops. How many different one-way tickets are possible?

7) How many different ways are there to arrange 8 identical large chairs and 3 identical small chairs in a row?

8) What is the probability of randomly, but correctly, guessing the top three finishers in a 20-person race?

9) Two dice are rolled. Find the probability that...
  a) the sum of the numbers showing on the dice is 5.
  b) the sum of the numbers showing on the dice is 11.
  c) you get a 6 on exactly one die.
  d) you get a 6 on at least one die.

10) A bag has 2 red, 4 pink, and 6 blue marbles in it. Two marbles are drawn at random. Find the probability that...
  a) both are red.
  b) one is red and one is pink.
  c) neither is red.

11) Five coins are tossed. Find the probability of …
  a) getting all heads?
  b) getting one head?
  c) getting two heads?
  d) getting at least three heads?

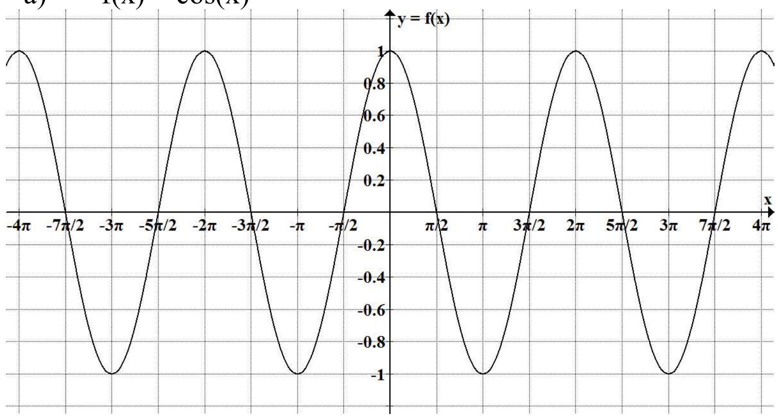
12) What is the probability of drawing a red card or a 5 from a standard 52-card deck?

13) Ten identical coins are to be distributed randomly between four people. How many ways can this be done?

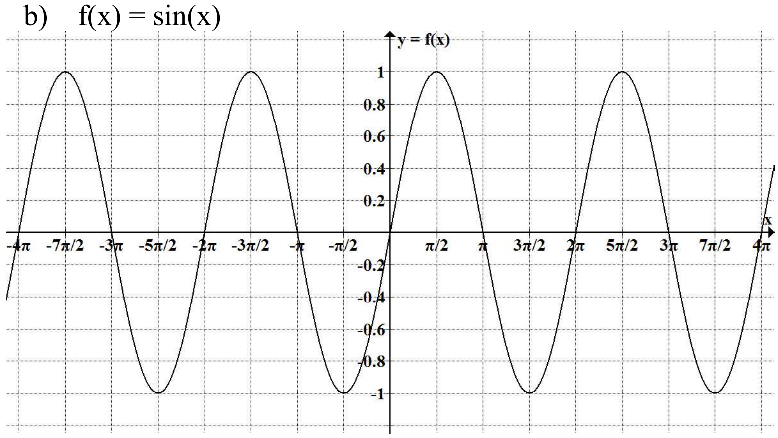
14) With a 13-card hand, what is the probability of getting…
  a) only red cards?
  b) no face cards (J, Q, K)?
  c) at least one face card?
  d) one card of each kind (one ace, one king, etc.)?
  e) exactly 11 diamonds?

6)

a) f(x) = cos(x)



b) f(x) = sin(x)



c) f(x) = tan(x)