Keep it alive!

Ŵorking on skills is important, but all too often skills dominate everything and then the love of the subject is lost. So how do we keep the class enthusiastic about learning math as they are learning all these skills? This is the great balancing act. Here are some ideas:

- <u>You are the author!</u> In order to make a lesson come alive, the teacher needs to penetrate the material, make it their own, and in the end become the author of what is presented to the students.
- <u>Working in groups</u>. Rather than having the teacher at the board explaining one problem after the other, have the students try to figure out some of the more challenging problems in groups.
- <u>Looking ahead</u>. Try looking over the entire unit before it begins and note the key conceptual steps. Prepare the class for what is to come, and find ways to get them excited about it.
- <u>Mathematical experiences</u>. Sometimes the class needs something different for a day or two. Some ideas for this might be: going outside for a math related activity, working on a math puzzle (usually in groups), or playing a math game. Our puzzle and game book is a great resource for this.

Homework and pace

- <u>Pacing</u>. How much of this workbook a teacher can cover during the year will depend upon the teacher, the class, and the amount of time available. What appears in this workbook is literally the entire year of math assignments for my eighth grade class. It's rare that I get through it all. On average, my eighth graders are able to work through one or two assignments per week each one a "practice sheet" or a "Group Sheet". This assumes a well-prepared class coming into eighth grade. Don't rush through the material for the sake of "getting through it." It may be better to cover less material more thoroughly.
- <u>Homework</u>? Teachers often feel compelled to give homework to their students. But what is the value of homework? Does it truly help their understanding and advance their skill level? Does it increase their enthusiasm for learning? I find that the students who struggle the most with math are the least capable of doing their math work at home. And relying on parents to help their children with homework is often counter-productive. Additionally, homework causes many students to dislike math. I used to give my middle school students a fair bit of homework (work to be done at home); I now try to give no "homework". Instead of requiring my students to do math work at home, I have added one math period per week to their schedule, which is set aside for the purpose of "homework". This approach has been more successful for many reasons, one of which is that I am present to answer the students' questions.
- <u>Different needs</u>. There is a range of abilities in any class. Yet, I believe strongly that math classes should not be divided into slower and faster-paced sections until ninth grade. For some students, the assignments are quite challenging. Sometimes, struggling students may need an IEP (Individualized Education Program) to ensure that their needs are met, which may include modified expectations and reduced workload.
- <u>Carefully choose your assignments</u>. Much of the material found in this workbook is quite challenging. The teacher's job is a balancing act challenging the students, yet making sure they are not overwhelmed. Assigning all of the problems from a given sheet is often not the best practice. Instead, the teacher should first carefully read through a given sheet, and then decide which problems to assign, which ones to leave out, and which ones to leave as challenge problems. One method is to assign "normal" problems (which everyone is expected to do), "red" problems (the challenging problems that only certain students are required to do), and "green" problems (the more basic problems that aren't required for those students doing the "red" problems).
- <u>Corrections</u>. I do not collect and correct assignments. My main reasons for this are: (1) personal sanity; (2) students should be able feel it is OK to make mistakes; and (3) students need to become responsible for their own corrections.

The seven units in this workbook are:

- <u>Number Bases</u>. This unit is a wonderful way to challenge students and develop real mathematical thinking. Number bases can be part of a main lesson.
- <u>Pythagorean Theorem</u>. Building upon what was begun in seventh grade, this relatively brief unit thoroughly explores the Pythagorean Theorem. It works with the square root algorithm, which should be introduced before beginning the worksheets in this unit.
- <u>Mensuration</u>. "Mensuration" means the study of measurement. In eighth grade, we focus on areas and volumes. Mensuration can be part of a main lesson.
- <u>Percents & Growth</u>. I introduce percents in sixth grade, cover the basics in seventh grade, and then to go into depth in eighth grade. If done well, the students will enter high school having a firm grasp of percents.
- <u>Proportions and Dimensional Analysis</u>. This is the most challenging unit in the book. It builds upon what was introduced during the *Measurement* unit and *Ratios* unit in seventh grade. *Dimensional Analysis* consists mostly of changing units (e.g., "58 pounds is how many kilograms?"). This unit also includes the topics of *Density* and *Rates*. There are a few questions that involve Archimedes' principle, which ought to be covered in the physics main lesson before beginning this unit.

Number Bases – Group Sheet #6

 Convert to decimal. (There is a trick that makes them all easy!) a) 77_{oct} b) 777_{oct} c) 7777_{oct} d) 7777_{oct} e) 444_{five} f) 44444_{five} 	 3) Assume that you need to invent a type of Morse code where each character (e.g. a letter) is represented by a certain number of long or short beeps. Example: How many different characters can be represented if there are 2 beeps per character? Answer: 4. (The 4 different characters are long-long, long-short, short-long, short-short.) 	5) With computers, we speak of <i>bits</i> instead of "beeps". A bit can be thought of as a switch. Instead of hearing a long or short beep, a computer "sees" whether each bit is "on" or "off". A <i>byte</i> is simply a collection of eight bits, which is used to represent one character. A <i>kilobyte</i> is 2^{10} (≈ 1000) bytes. A <i>megabyte</i> is 2^{20} ($\approx 1,000,000$) bytes. A <i>gigabyte</i> is 2^{30} ($\approx 1,000,000$,000) bytes.
g) FF _{hex}	How many different characters can be represented if there	
 h) FFF_{hex} i) FFFF_{hex} 	area) 3 beeps per character?	a) How many different characters can be represented by one byte?
j) 111 _{bin} k) 1111 _{bin}	b) 4 beeps per character?	b) How many bytes of computer memory are
l) 11111 _{bin} m) 111111111 _{bin}	c) 5 beeps per character?d) 6 beeps per character?	(Given 6 characters per word, 15 words per line, and 50 lines per page.)
 2) Calculate in the base indicated. a) (5_{oct})³ 	e) 7 beeps per character?	c) Your above answer is equal to how many kilobytes?
	f) 8 beeps per character?	d) One gigabyte is enough to store about how many pages of
b) $\sqrt{10_{\text{hex}}}$	4) What is the minimum number of beeps per character that would be needed in order to represent all the letters and	plain text?
d) $\sqrt{100_{hex}}$	aigits?	

Mensuration – Practice Sheet #3



Solutions to Selected Problems

Mensuration – Practice #1

4) a)
$$D \approx \frac{22}{7} \cdot 35 \rightarrow \underline{110m \text{ or}} D \approx 3.14 \cdot 35 \rightarrow \underline{109.9m};$$

 $D \approx 35 \div \frac{7}{22} \rightarrow \underline{110m \text{ or}} D \approx 35 \div 0.318 \rightarrow \underline{110.1m}$
b) $D \approx \frac{22}{7} \cdot 3 \rightarrow \underline{9.43m \text{ or}} D \approx 3.14 \cdot 3 \rightarrow \underline{9.42m};$
 $D \approx 3 \div \frac{7}{22} \rightarrow \underline{9.43m \text{ or}} D \approx 3 \div 0.318 \rightarrow \underline{9.43m}$
c) $D \approx \frac{7}{22} \cdot 44 \rightarrow \underline{14m \text{ or}} D \approx 0.318 \cdot 44 \rightarrow \underline{14m};$
 $D \approx 44 \div \frac{22}{7} \rightarrow \underline{14m \text{ or}} D \approx 44 \div 3.14 \rightarrow \underline{14.01m}$
d) $D \approx \frac{7}{22} \cdot 20 \rightarrow \underline{6.36m \text{ or}} D \approx 0.318 \cdot 20 \rightarrow \underline{6.36m};$
 $D \approx 20 \div \frac{22}{7} \rightarrow \underline{6.36m \text{ or}} D \approx 20 \div 3.14 \rightarrow \underline{6.37m}$

- 5) d) From the midpoint of one of the slanted sides, imagine rotating it so it is vertical, creating a 90° angle. Do the same the other slanted side and this creates a rectangle with the same area having a height of 4 in. and a base equal to 10, which is the average of the top and bottom of the trapezoid. Now do $B \cdot H \rightarrow$ $10.4 \rightarrow 40 \text{ in}^2$; **OR** The area of the rectangle with the top as 7 and the bottom as 7 is 28 sq.in The area of the triangle on the left is $\frac{1}{2}B \cdot H \rightarrow 1.5 \cdot 4 \rightarrow 6$ and the area of the triangle on the right is also 6. 28+6+6 \rightarrow 40 in²; **OR** Draw a line from the top right corner to the bottom left corner, thereby creating two triangles. The top triangle has a height of four and a base of 7 (maybe easier if you imagine flipping the triangle flipped) Therefore the area of that triangle is $\frac{1}{2}B \cdot H$ (take half of H instead cause the number is easier) $B^{1/2}H \rightarrow 7^{1/2}(4) \rightarrow 14$. The bottom triangle has a height of 4 and a base of 13. The area of this triangle is $B^{1/2}H \rightarrow 13^{1/2}(4) \rightarrow 26.14 + 26 \rightarrow 40 \text{ in}^2$
 - e) Find the height of the triangle. $\sqrt{c^2-a^2} = b^2 \rightarrow \sqrt{10^2-5^2} \rightarrow \approx 8.66$; Area of half of the triangle is $\frac{1}{2}B \cdot H \rightarrow \frac{1}{2} \cdot 5 \cdot 8.66 \rightarrow 21.65 \rightarrow$ multiply by $2 \rightarrow \underline{43.3cm^2}$

Mensuration – Practice #2

- 1) b) $3.14 \cdot 4^2 \rightarrow 16\pi \approx 50.24 \text{ cm}^2$
- 2) b) $A = \frac{1}{2}B \cdot H \rightarrow \frac{1}{2} \cdot 11 \cdot 60 \rightarrow \underline{330cm^2}$
 - d) $A = \frac{1}{2}B \cdot H \rightarrow \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} \rightarrow \frac{3}{16} \frac{in^2}{in^2}$ e) $a = \sqrt{c^2 - b^2} \rightarrow \sqrt{15^2 - 12^2} \rightarrow 9;$ $A = \frac{1}{2}B \cdot H \rightarrow \frac{1}{2} \cdot 9 \cdot 12 \rightarrow 54m^2$
 - f) $A = \frac{1}{2}(top + bottom) \cdot H = \frac{1}{2}(5.5 + 8.5) \cdot 4 = \frac{28m^2}{28m^2}$ <u>OR</u> The area of the rectangle is $B \cdot H = 22$, The area of the triangle is $\frac{1}{2}B \cdot H = \frac{1}{2} \cdot 3 \cdot 4 = 6$ Total area = $22 + 6 \rightarrow \frac{28m^2}{28m^2}$
 - h) Start by slicing the triangle in half with a vertical line. $H = \sqrt{12^2 6^2} \rightarrow \approx 10.39$; $A = \frac{1}{2}B \cdot H \rightarrow A = \frac{1}{2} \cdot 12 \cdot 10.39 \approx 62.4 \text{ cm}^2$
- 3) c) Base is the triangle. $V = A_{Base} \cdot H = 30 \cdot 10 = 300$

Mensuration – Group #2

1) b) $A_{\text{Base}} = \pi \cdot 8^2 \approx 64\pi; \quad V = \frac{1}{3} \cdot A_{\text{Base}} \cdot H;$ $V = \frac{1}{3} \cdot 64\pi \cdot 12 = 256\pi \approx 804 \text{ cm}^3$ 2) a) Surface Area: Area of the base rectangle = 80; Area of each side rectangles = 50;Area of each triangle = 12; (Note the height of the triangle is 3cm.) Total surface area = $80 + 2 \cdot 50 + 2 \cdot 12 \rightarrow 204 \text{cm}^2$ Volume: Imagine placing the prism on a table such that the triangle is the base. Area of triangle is 12 $V = A_{Base} \cdot H = 12 \cdot 10 = 120 \text{ cm}^3$ b) Surface Area: There are three pieces here: the top and bottom circles, and the tube, which unfolds as a rectangle. Area of the circles = $\pi 2.5^2 = 6.25\pi$ Area of the tube = $6 \cdot 5\pi = 30\pi$ Surface Area = $2 \cdot 6.25\pi + 30\pi = 42\frac{1}{2}\pi \approx 133.5$ Volume: $A_{Base} = \pi 2.5^2 = 6.25\pi$

$$V = A_{Base} \cdot H = 6.25\pi \cdot 6 = 37\frac{1}{2}\pi \approx \frac{118 \text{ ft}^3}{2}$$

3) a) $12^2 = \underline{144}$ b) $12^3 \rightarrow \underline{1728}$

Mensuration – Practice #3

- 4) b) Try to avoid just giving a formula for this one. See how many different ideas students can think of for finding the area of this trapezoid.
 - e) Start by slicing the triangle in half with a vertical line. $H = \sqrt{c^2 b^2} \rightarrow \sqrt{3^2 1.5^2} \rightarrow \approx 2.60;$ $A = \frac{1}{2}B \cdot H \rightarrow \frac{1}{2} \cdot 3 \cdot 2.60 \rightarrow \approx 3.90m^2$

5) a)
$$8'' = \frac{2}{3}$$
 ft $V = B \cdot H \cdot L \rightarrow 3 \cdot \frac{2}{3} \cdot 2 \rightarrow \frac{4 \text{ ft}^3}{4 \text{ ft}^3}$

- b) $A_{\text{Base}} = \pi \cdot 3^2 = 9\pi$ $V = A_{\text{Base}} \cdot H = 9\pi \cdot 3 = 27\pi \approx \mathbf{\underline{84.8cm}^3}$
- c) $A_{Base} = \pi \cdot 4^2 = 16\pi$ $V = \frac{1}{3} \cdot A_{Base} \cdot H = \frac{1}{3} \cdot 16\pi \cdot 4 = \frac{64\pi}{3} \approx \frac{67.0 \text{ m}^3}{3}$
- e)

As shown in the drawing here, we imagine two triangles, one of which is inside the pyramid and has a height equal to the height of the pyramid. The left triangle is a Pythagorean triple, which tells us that x is 40. Then we use this value for x with the second triangle to get: $H^2 = 40^2 - 30^2 \rightarrow H \approx \sqrt{700} \approx 26.46$ Now, $V = \frac{1}{3} \cdot A_{Base} \cdot H$ $V = \frac{1}{3} \cdot 3600 \cdot \sqrt{700} \approx 31,750m^3$

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Solutions to Selected Problems

Percents – Practice #6

- 1) $230 \cdot 1.028 =$ <u>\$236.44</u>
- 2) $53 \div 66 \approx 80.3\%$
- 3) % increase = $\frac{\text{amount of increase}}{\text{starting point}} = \frac{41-17}{17} \approx 1.41$, which is a **141% increase**
- 4) Reword as: "76.26 is 123% of what?" This is also a reverse problem, so we divide $76.26 \div 1.23 = \underline{62}$
- 5) This is a reverse problem, so we divide $162 \div 0.036 = 4500$
- 6) Here are two ways to solve this: % decrease = $\frac{\text{amount of decrease}}{\text{starting point}} = \frac{395-158}{395} = 60\%$ or, 158 ÷ 395 = 0.40, which means "40% of", or a <u>60% decrease</u>
- 7) 8200 decreased by 93% is the same as 7% of 8200. $8200 \cdot 0.07 = 574$
- 8) Reword as: "69 is 89% of what?" This is also a reverse problem, so we divide $69 \div 0.89 \approx \underline{77.5}$
- 9) $2.65 \cdot 4300 = \underline{11,395}$
- This is the same question as given in the previous problem. Answer: <u>11,395</u>
- 11) 8÷53 ≈ <u>15.1%</u>
- 12) The answer for this will be 10 times smaller than in the previous problem because here the denominator is 10 times greater. Answer: <u>1.51%</u>
- 13) Here are two ways to solve this: % increase = $\frac{\text{amount of increase}}{\text{starting point}} = \frac{737-536}{536} = 37.5\%$ or, 737÷536 = 1.375, which means "137^{1/2}% of", or a 37^{1/2}% decrease
- 14) This is a reverse problem, so we divide $6794 \div 0.79 = \underline{8600}$
- 15) Reword as: "61.56 is 162% of what?" This is also a reverse problem, so we divide $61.56 \div 1.62 = 38$
- 16) Here are two ways to solve this: % decrease = $\frac{\text{amount of decrease}}{\text{starting point}} = \frac{320-307.2}{320} = \frac{4\%}{6}$ or, 307.2÷320 = 0.96, which means "96% of", or a 4% decrease
- 17) Increase by 25% means "125% of", and Decrease by 25% means 75% of". Therefore: 400 · 1.25 · 0.75 = <u>375</u>

- 18) $72 \div 2.4 \approx 30$ years
- 19) 72÷18 ≈ <u>4%</u>
- 20) We ask: "572.94 is 6.1% more than what?" Reworded as: "572.94 is 106.1% of what?" This is also a reverse problem, so we divide $572.94 \div 1.061 = 540$
- 21) a) $74 \div 185 = 40\%$
 - b) 185 ÷ 74 = <u>250%</u>
 - c) The question is: "going from 185 down to 74 is what percent decrease?" Since 74 is 40% of 185, we can say that 74 is <u>60% less than</u> 185.
 - d) The question is: "going from 74 to 185 is what percent increase?" Since 185 is 250% of 74, we can say that 185 is **150% more than** 74.
- 22) a) Reword as: "Bob weighs 120% of Pete" 180 · 1.2 = <u>216 lb</u>
 - b) The question is: "180 is what % less than 216?" $180 \div 216 = 83\frac{1}{3}\%$, which is <u>16²/₃% less</u>.
- 23) a) Reword as: "Mike weighs 80% of TJ" This is also a reverse problem, so we divide $180 \div 0.8 = \underline{225 \, lb}$
 - b) The question is: "225 is what % more than 180?" $225 \div 180 = 125\%$, which is **25% more**.
- 24) This is a reverse problem, so we divide $252 \div 0.35 = \$720$
- 25) a) $P = 127 (1.0023)^{10} \approx 130 \text{ million}$
 - b) $P = 127 (1.0023)^{80} \approx 153 \text{ million}$
 - c) $P = 123 (1.026)^{10} \approx 159$ million
 - d) $P = 123 (1.026)^{80} \approx 959$ million
 - e) 2.6% annual population growth is extremely high for a country and is unlikely to continue for very long.
- 26) Depreciation is really negative growth. The formula is then: $P = 12,000 (1 - 0.15)^8 \approx \3270