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—— *Unit-by-Unit Commentary* ——

The units covered in this workbook are:

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Pre-Algebra

This unit is only intended for a class with weak basic pre-algebra skills. See *Prerequisites* (above) in order to determine if a class is prepared enough to be able to skip this unit and move directly into the *Algebra Basics* unit. If you decide that a class needs this *Pre-Algebra* unit, then here is a word of advice: don't spend more than three weeks on it. There will certainly be more opportunities to further strengthen these basic skills in the units to come.

Algebra Basics

Overview

Essentially, this unit is a very thorough review and deepening of the topics found in the algebra unit of our eighth grade workbook. Conceptually, this unit should not be too challenging for the students. Many of the problems in this unit require a great many steps. By practicing such problems, hopefully the students will become more skillful and eliminate their "silly" mistakes. A decent mastery of this unit is needed before moving forward in this workbook.

Evaluating expressions

These problems are deceptively difficult for many students. They demand an extra degree of care, and often include some traps. As an example, consider this:

Evaluate the following expression given that $x = 3$; $y = -2$.

$$y^2 - xy + 4 - \left(\frac{x}{y}\right)^3$$

(Solution: $17\frac{3}{8}$)

This problem presents many challenges for the typical student, including getting the signs correct, and dealing with fractions.

formula. We introduce the method of completing the square in Problem Set #3, and then the students practice this method in each of the next five problem sets.

Through the process of working with completing the square we encounter absolute values – specifically the idea that if we are taking the square root of something squared then it is the equivalent of the absolute value of that something. In other words: $\sqrt{x^2} = |x|$

At first, the students' work for completing the square looks like this:

$$x^2 + 10x - 24 = 0$$
$$x^2 + 10x = 24$$
$$x^2 + 10x + 25 = 24 + 25$$
$$(x+5)^2 = 49$$
$$\sqrt{(x+5)^2} = \sqrt{49}$$
$$|x+5| = 7$$
$$x+5 = \pm 7$$
$$x = -5 \pm 7$$
$$x = -12, 2$$

The arrow indicates that after a while the students can take a shortcut, realizing that whenever they take the square root of something squared that a plus-minus sign needs to appear on the other side of the equation.

The students are then expected (perhaps working in small groups) to derive the quadratic formula for themselves in Problem Set #9. This may be the most advanced, complicated, and rewarding mathematics that the students experience all year.

Logarithms

Overview

We feel that students graduating high school should have a firm understanding of logarithms. Therefore, logarithms appear as a unit in each of our three workbooks. The first unit here, in our ninth grade workbook, is kept quite simple. This is intentional. We want to be sure that this important foundation is thoroughly understood by every student. It then is allowed to "sleep" in the student until tenth grade, at which point it is reviewed and deepened, and then revisited and deepened again in eleventh grade when things become much more complex.

We realize that this unit falls towards the end of the year, and that there may not be adequate time to work through the whole unit. However, even if time is running short, it is important to spend at least two or three days giving a brief introduction to the idea of logarithms. This short time is a valuable seed planted for a deeper study of logarithms in the next two years.

Factoring

Teachers: Read the commentary on this unit in the introduction.

Problem Set #1

Group Work

In mathematics, factors of a given number are those numbers that go into it evenly. We can also *factor* a number into a product of its factors. For example: $35 \rightarrow 5 \cdot 7$

In algebra, we can also factor polynomials. For example, we can multiply $3x^4(5x + 6) \rightarrow 15x^5 + 18x^4$, or do the reverse and factor $15x^5 + 18x^4 \rightarrow 3x^4(5x + 6)$.

Multiply.

1) $7(4x - 3)$
2) $x^3(x^2 - 5)$
3) $3x^2(2x^3 + 7)$
4) $5x^2y^3(4x - 3y)$

Factor.

5) $28x - 21$
6) $x^5 - 5x^3$
7) $6x^5 + 21x^2$
8) $20x^3y^3 - 15x^2y^4$
9) $15x^4 + 25$
10) $7x^8 + 10x^5$
11) $12x^6 - 8x^5 + 20x^4$

Homework

Section A

Multiply.

12) $7(2x + 5)$
13) $5x^2(3x - 4)$
14) $6y^4(5y^3 + 3)$
15) $x^6(x^2 - 3x + 11)$

Factor. (Then multiply, in order to check your answer.)

16) $14x + 35$
17) $15x^3 - 20x^2$
18) $30y^7 + 18y^4$
19) $x^8 - 3x^7 + 11x^6$
20) $10x^4 - 15$
21) $y^8 - 4y^5$
22) $x^5 - 13x^4 + 6x^3$
23) $4x^7 + 12x^6 - 32x^5$

Section B

Multiply.

24) $(4x^2y^3)(2xy^3)$
25) $(4x^2y^3)(2x + y^3)$
26) $(4x^2 + y^3)(2x + y^3)$
27) $(5x^3y^4)^2$
28) $(5x^3 + y^4)^2$
29) $(5x^3 - y^4)(5x^3 + y^4)$
30) $(2x^3 - y^4)(5x^3 + y^4)$

Factor. (Then multiply, in order to check your answer.)

31) $12x^3y^5 + 8x^4y^4$
32) $9x^4y - 3x^3y^2 + 6x^2y^3$
33) $x^5 - 2x^2$
34) $10x^5 - 2x^2$

Note: The factoring that appears on this sheet is called *factoring out the GCF (greatest common factor).*

Logarithms – Part I

Teachers: Read the commentary on this unit in the introduction.

This unit is a brief introduction to an important topic. In the next two workbooks, logarithms are revisited (and covered in more depth) in *Logarithms, Part II* and *Logarithms, Part III.*

Problem Set #1

Simplify.

1) $(x^3)^2$
2) $x^3 \cdot x^2$
3) $(x^6)^4$
4) $x^6 \cdot x^4$
5) x^3+x^2
6) x^4+x^4
7) $(4x^3)^5$
8) $3x^4 \cdot 7x^5$
9) $(3x^5)^4$
10) $2x^3 \cdot 3x^9$
11) $4x^3+2x^5$
12) $2x^3+5x^3$
13) $3x^3-x^3$

Give a simplified answer that has no negative exponents.

14) $x^{-4}x^7$
15) $\frac{x^{-4}}{x^3}$
16) $5x^{-4}$
17) $3x^2y^{-5}$
18) $3y^{-5}x^2$
19) $3y^{-5} + 4x^2$
20) $(\frac{2}{3})^4$
21) $(\frac{2}{3})^{-4}$
22) $(\frac{2}{3})^{-1}$
23) $(\frac{2}{3})^0$
24) $(x^{-4})^5$
25) $(3x^{-3})^2$
26) $(9x^{-4}y^3)^4$
27) $(6x^{-3}y^7)^3$
28) $(6x^{-6}y^2)^3$
29) $(4x^{-6}y^2)^{-4}$
30) $(8x^6y^{-5})^{-3}$
31) $\left(\frac{x^{-3}}{y^2}\right)^3$
32) $\frac{15x^{-4}y^{-3}}{6x^{-7}y^0}$
33) $\frac{8x^{-4}y^7}{6x^3y^3}$
34) $\left(\frac{8x^{-4}y^7}{6x^3y^3}\right)^{-1}$
35) $\left(\frac{8x^{-4}y^7}{6x^3y^3}\right)^2$
36) $\left(\frac{8x^{-4}y^7}{6x^3y^3}\right)^0$

Calculate each. Use the *Power and Base Tables* (on the next page), if needed. Leave square roots in simplified form.

37) 8^2
38) 8^{-2}
39) 6^0
40) 6^{-1}
41) 6475^{-1}
42) 7384^0
43) 7^{-3}
44) $\log_3 9$
45) $\log_2 16$
46) $\log_5 25$
47) $\log_5 625$
48) $\log_4 64$
49) $\log_2 64$
50) $\log_8 64$
51) $\log_2 1024$
52) $\log_9 531441$
53) $\log_7 16807$
54) $\log_2 2$
55) $\log_2 1$
56) $\log_{57} 57$
57) $\log_{29} 1$
58) Explain why each one is true:
   a) $2^{10} = 4^5$
   b) $3^{10} = 9^5$
   c) $2^9 = 8^3$
   d) $4^6 = 8^4$
59) A rectangle's length is 4 more than three times its width. Find its length if the perimeter is 52.

Possibility & Probability – Part I

This unit is intended to review and deepen material that has been introduced in main lesson. This topic is covered one more time in eleventh grade with *Possibility & Probability, Part II.*

Teachers: Read the commentary on this unit in the introduction.

Problem Set #1

Section A

1) Bob's Bikes makes bikes with 2 types of frames, 3 handle bar styles, and in colors red, yellow, green, black, or white. How many different bikes can they make?

2) Paul's Pizza offers 3 choices of salad, 20 kinds of pizza, and 4 different desserts. How many different 3-course meals can be ordered?

3) How many 7-digit phone numbers are possible? (The first digit cannot be 0 or 1.)

4) In how many ways can 8 people be lined up in a row?

5) A license plate consists of 3 letters followed by 3 digits (e.g., XBB022). How many different plates could be issued?

6) How many ways can four different roles in a play be assigned from a group of 14 actors?

7) In how many ways can a president and a secretary be chosen from a group of 6 people?

8) In a 6-horse race…
   a) how many different orders of finishing are there?
   b) how many possibilities are there for the first 3 places?

9) A character can be either a letter or a digit. (Thus, there are 36 different characters.)
   a) How many possible three-character codes are there?
   b) How many possible three-character codes have different characters and a digit as the first character?

Section B

10) How many license plates are possible that have 2 digits and 2 letters (in any order)?

11) Using the letters of the word EQUATION, how many 4-letter words (which don't have to spell anything) can be formed (without repetition)…
   a) that start with T?
   b) that start and end with a consonant?
   c) that have only one vowel?
   d) with all the vowels positioned furthest to the right?

Problem Set #4

1) 16
2) $\frac{53}{3} = 17\frac{2}{3}$
3) -3
4) 48
5) 16X – 35
6) 0
7) 197
8) 59
9) No solution.
10) X = 0
11) All real numbers.
12)
    a) X = 7
    b) X = $-\frac{1}{7}$
13)
    a) X = $-\frac{15}{7} = -2\frac{1}{7}$
    b) X = $\frac{10}{7} = 1\frac{3}{7}$
14)
    a) X = 4
    b) X = $\frac{4}{9}$
15)
    a) X = $\frac{9}{4} = 2\frac{1}{4}$
    b) X = $-\frac{13}{6} = -2\frac{1}{6}$
16)
    a) X = $\frac{3}{2} = 1\frac{1}{2}$
    b) X = $-\frac{7}{8}$
    c) X = $-\frac{5}{14}$
17)
    a) X = $\frac{4}{11}$
    b) X = 3
18) X = 9
19) X = $\frac{22}{85}$

Problem Set #5

1) 31
2) 11
3) 9
4) -60
5) -25
6) $\frac{139}{8} = 17\frac{3}{8}$
7) All real numbers.
8) X = 0
9) No solution.
10)
    a) X = $-\frac{2}{15}$
    b) X = $\frac{34}{5} = 6\frac{4}{5}$
11)
    a) X = $\frac{3}{14}$
    b) X = $\frac{44}{21} = 2\frac{2}{21}$
12)
    a) X = 13
    b) X = $-\frac{3}{2} = -1\frac{1}{2}$
13)
    a) X = $\frac{13}{16}$
    b) X = $\frac{2}{3}$
14) X = 2
15) X = $\frac{65}{147}$
16) X = $\frac{17}{12} = 1\frac{5}{12}$

Problem Set #6

1) X = -5Y + 4
2) X = $\frac{3}{4}$Y
3) X = 3Y – 3