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–Thoughts on Teaching Lower Grades Math–

The challenge of teaching math in the lower grades

Fear and stress

In this day and age, fear can have an influence over much of our lives, including our children’s education. Teachers and parents are more worried than ever. “What will happen if my child gets behind?” The pressure to get ahead has increased. Education has become a race! More advanced material at a younger age. More homework. More high stakes testing. More fear. More stress. Even independent schools are affected.

Stress and fear are not conducive to learning anything; they impair our students’ ability to learn math. We need to reduce fear and stress and create a calm, safe learning environment, which enables our students to achieve an inner state of calm contemplation. This creates a vessel in which learning math is possible.

Avoiding struggle

Ironically, even though there are greater expectations for our children to learn more material and do more homework – all at a younger age – we don’t want them to struggle. We want our children to be happy and successful. We live in an “instant gratification” culture. If we want something to happen, we expect it quickly. And it should be easy. Our children should get A’s, be ahead of the others, and be comfortable.

The trouble is that learning math usually isn’t that quick and easy. It takes patience – patience for the student, patience for the teacher, and patience for the parents. *Working through struggle is an important part of learning math – and is a great life lesson.* One of our challenges as class teachers today is to create a safe environment – without fear and stress – in which our students can work through their struggles. This is certainly no easy task. Wouldn’t it be wonderful if all of our children could learn how to calmly face their challenges and develop confidence as they successfully work through their struggles?

What are the symptoms of our “math illness”?

Meaningless math

What is society’s view of math? Math is often not seen as meaningful in and of itself. A widespread belief is that “good math” must be practical and useful. It then follows that the main reason to study math is that it can be useful for some other purpose or subject. All of these attitudes toward math can make it challenging to teach math in a meaningful way.

While we, the authors, believe that practical applications of math can be helpful, we also feel that there is a higher purpose to learning math. (See *Principles of Waldorf Education* below.)

The LIST

If people believe that math should always be practical, then it stands to reason that skill development would become the primary focus. Over time this has degenerated into the *LIST* – a horrifically long list of skills that are supposedly necessary for our children to move to the next level (e.g., middle school, high school, or college). The *LIST* manages to put pressure on teachers at all levels of education. They fear that if they don’t get through the entire *LIST*, then their students won’t be prepared. We are led to believe that this *LIST* is getting longer and longer, and more and more daunting. Such an over-emphasis on skill development is boring or overwhelming (or both) for many students, and for the teacher as well.

We, the authors, believe that basic skills are important, but the list of topics needed to move forward is not horrifically long; it is actually quite manageable (see *What skills are really needed?* below). We also believe there is much more to math than just learning skills. With so much emphasis going toward the mastery of a long list of skills, some of the more interesting aspects of mathematics become neglected. (See *Principles of Waldorf Education* below.)

Making math procedural

The tendency in schools today is to spend far too much time on procedural skills at too young an age. Examples include borrowing in second grade, long division in third grade, and arithmetic with mixed numbers in fourth grade. Each of these topics would probably be better introduced at least a full year later than what was just mentioned. But here’s the real issue: by having too much of an emphasis on procedural skills at too young an age, the children experience math as a collection of blind procedures, likely paving the way for math anxiety or trauma. Even for those students who aren’t traumatized by the blind procedures, it is likely that they will not

–Second Grade Math–

Overview of Child Development in Second Grade

The second grader is in the middle of an important developmental phase between the ages of six and nine. It is a golden time where the student is still king in his own kingdom. The second grader is now livelier and more aware of what is happening around him. He may even become mischievous. Physically, the roundness of the early childhood years has disappeared. The mouth is closed and the student becomes less dreamy. The child's personality and character become more evident. We can help the children to overcome the one-sidedness of their character by telling fables and saint stories. The joy of learning is based on strong habits, rhythms, and songs. The creative forces of the child increase along with an ability to create more vivid inner pictures. The second grader's confidence grows and builds upon the foundation laid in grade one. However, the student still needs strong leadership from the teacher through a consistent and rhythmical approach to the lessons. An artistic approach is used to awaken the intellect.

With the math lessons, this is the year to introduce the times tables – all of them brought in an artistic way. The child is eager to show his individual abilities, such as the joy of finding missing numbers, or the discovery of geometrical patterns in the times tables. At this age, the child's memory forces are very strong, so it is an ideal time to begin to learn the arithmetic facts.

Curriculum Summary for Second Grade Math

The world of numbers

- The children should become fluent with counting up to 100. They should be able to start anywhere and continue counting without any hesitation, and do this either counting forward or backward.
- The students should gradually become at ease with the number world up to 1000 (and possibly higher).
- *Estimating.* We build up from the estimating done in first grade and progress to more challenging estimations.

Place value. Place value is introduced and practiced. Be sure that all the children really get this; this is an important step needed in order to deeply understand the world of numbers.

Addition and subtraction facts. By the end of the year, the class should have learned their addition facts (up to 24) and corresponding subtraction facts *by heart*.

The times/division tables

- Beginning in second grade, multiplication and division go together. The times tables and division tables are really the same thing.
- By the end of the year, the class should be comfortable with all of the times and division tables (from 1-12), in a row. This requires daily, systematic work! Be sure to read *A Step-by-Step Progression for the Arithmetic Facts* (in the appendix).

The four processes

- *Addition.* By the end of the year, the class should be *at ease* with adding any two-digit number with a one-digit number (e.g., $57+6$).
- *Subtraction.* By the end of the year, the class should be *at ease* with subtracting any one-digit number from a two-digit number (e.g., $52-6$), and also with subtracting two 2-digit numbers such that the answer is a one-digit number (e.g., $72-69$).
- The students must gain a deeper understanding of the concept of multiplication and division, such as:
 $3 \times 2 = \underline{\quad}$ asks: "Three groups of two makes what?"
 $12 \div 3 = \underline{\quad}$ asks: "How many groups of three fit in twelve?"
- By the end of the year, the children should be able to do all four processes (even alternating on the same page) and know the difference between the processes without help (but keep the problems simple!).
- The introduction to both vertical addition (i.e., carrying) and vertical subtraction (i.e., borrowing) should wait until third grade. Therefore, all written work with the four processes should still be done in horizontal form (e.g., $9=5+4$, $5+4=9$).

Time orientation. The children should become at ease with the days of the week, the months of the year, and terms like "tomorrow", "yesterday", "noon", "afternoon", "evening", "six o'clock", etc.

Math Main Lesson Block #2: Measurement

New material and content

- *Measurement.* This is the main theme of the main lesson.
 - *Linear measurement.*
 - The children are introduced to linear measurement through hands-on experiences based on the human body.
 - They should measure things in and around the classroom.
 - As a class we can make a simple graph that shows one particular student's height over a period of several months – or maybe each child could do that for himself at home.
 - *Weight.* Weight should also be part of the experience. The students can weigh objects in the classroom, and they can weigh one another.
 - *Volume and liquid measure.* With liquid measure we should pour cups into gallons, tablespoons into a cup, etc.
 - *Origins.* The historical origins of the various units of measurement can be brought in an imaginative way (e.g., through the story of Noah's Ark).
 - *Practical use.* Of course, it is best to show practical uses of measurement, such as with simple building projects, cooking recipes, etc.
 - *Estimation.* The students should practice estimating. For example, they could estimate the length of a board, the height of a tree, the distance between two rocks, the weight of an animal. We should always try to estimate before taking an actual measurement.
 - *Unit conversions.* The students should practice very simple unit conversions, such as converting feet to yards, feet to inches, pints to quarts, etc.
- *Money.* This is also the time to introduce the children to our currency. They can make their own money (coins and bills) and stock a store, or even create a market place. We can also create games for money. Through all of this activity, the children practice counting money and making change.
- *Vertical addition and subtraction.* Vertical addition and subtraction was introduced in the first math block of third grade. In the meantime, this new material has been “put to sleep” (see *The conscious use of forgetting* under *Principles of Waldorf Education* above). It is important that we not touch upon new material in between two math blocks. Now that the material (vertical addition and subtraction) has had its proper “rest”, we can review it and then deepen it.
- *Division with remainders.* This should be done using the same dividend but different divisors. For example, using the number 14, divide it by 1, 2, 3, etc. ($14 \div 1 = \underline{\quad}$; $14 \div 2 = \underline{\quad}$; $14 \div 3 = \underline{\quad}$; $14 \div 4 = \underline{\quad}$, etc.) After some time, the students will improve at figuring out the remainder. This is preparation for vertical (long) division.
- *Don't forget to review and practice!* Previously introduced material needs to be reviewed, practiced, and furthered. (See above *Practice and review of old material under Lessons and Topics for Third Grade Math.*)

Movement. Continue the movement activities that were done in the first block.

Working with manipulatives

- Of course, there are unlimited possibilities for hands-on experiences and activities during this measurement block. This should bring a lively spirit to the lessons and enthuse the students.
- Many props and visual aids are helpful for this block. For example, with volumes we could use milk cartons, jugs, bottles, spoons, cups, cans, etc.
- Our work with money and currency lends itself nicely to working with manipulatives.

Bookwork

- The children should write what they have experienced (e.g., one gallon is 4 quarts, etc.) in their main lesson books. They can make drawings (e.g., of a foot) as it applies to the units of measurement.
- Be sure to visit our website (www.JamieYorkPress.com) to see full-color pages from students' main lesson books.