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Thoughts on Teaching Math

What makes a good math student?

As teachers, we all hope our students will become good at math. But to realize this we need to fully understand what math is, and what it isn't (see *Blind Procedures*, above). So now we ask: What are the key attributes that enable a student to become good at math? Here is my short list:

- *Striving to understand deeply*. We want our students to understand the concepts they encounter. Good math students are never satisfied with going through a procedure without understanding what they are doing.
- *Asking good questions*. Good math students are curious, and wonder "what if…?" They question why something is true, and they become skillful at articulating questions.
- *Making mistakes*. Contrary to what many people think, mistakes are an important part of learning math. Good math students don't let mistakes discourage them. In fact, mistakes can motivate students to find the truth and make mathematical discoveries. We want our students to become comfortable with making mistakes, and to learn from their mistakes.
- *Attitude and work ethic*. This includes many things, such as: enthusiasm, determination, and discipline. Good math students persevere through their challenges; they are determined to succeed.

All of the above shows how our students learn many life lessons through studying mathematics. (And, yes, the same may be said about the proper teaching of other subjects as well.)

What makes a good math teacher?

Many class teachers feel under-confident in their own math skills, and, in some cases, have had traumatic experiences with math when they were in school. Often, this results in the teacher developing an antipathy towards math. However, if such a teacher can find a way to rise above his antipathy toward math, then that teacher may find joy in math, which can result in bringing wonderful math lessons to the students.

This is what I feel makes a good math teacher:

- *Enthusiasm for learning math*. For many teachers, this amounts to finding a new relationship to math. How wonderful it can be to find out that math can be interesting and rewarding!
- *Ability to present the material effectively*. This is the art of teaching.
- *Adequate preparation time for the math lessons*. With everything that is demanded of the class teacher, there often isn't enough time left to prepare adequately for the math lessons.
- *A healthy relationship to the students*. This helps to create a safe and comfortable learning environment.

Teaching the "Big Topics"

The "big topics" in middle school math are fractions, decimals, percents, ratios, and (simple) algebra. Our students should have these topics mastered before entering high school. There are two common mistakes made with these big topics.

The first is to do *too much too soon*. The topic may have been introduced in a wonderful and effective way, but if we build too much on the new foundation then many of the students may drown. The second mistake is *not enough follow-up and review*. This often happens with percents. It is introduced and practiced (perhaps too much!) in sixth grade, and then the students might never see it again.

So how should percents, for example, be done? It should be introduced in a wonderful sixth grade main lesson – not too much – and kept very simple. Then, one year later, the topic is reviewed and deepened – again being careful that it isn't too much. And then, once again, it is put to sleep. Now the stage has been set for going into depth in eighth grade. A similar three-step plan can be followed with any "big topic". If we want the students to learn something well and permanently, then we need to create a "dance" between introducing, deepening, practicing, sleeping, and reviewing.

Separation of form and number

It is helpful to think of *form* (pure geometry) as having its roots in the physical/material world, and *number* as having its roots in the non-physical world of pure thought. In education today, *form* and *number* are often blended together. This can lead to unnecessary confusion.

There certainly are times when it is appropriate and helpful to integrate numbers into a geometry topic. For example, geometric figures become associated with numbers and algebraic formulas in the study of measurement (e.g., areas and volume). However, we also need to find ways to have our students experience "pure geometry" without attaching formulas and numbers. Waldorf schools do this starting in first grade with

Seventh Grade

The importance of seventh grade

Seventh grade is an important year academically. This is the year when students start to develop abstract thinking (through algebra, physics, essay writing, etc.). It is relatively common for a student to enter seventh grade fairly weak in math, but then to "wake-up" during seventh grade, and, in the end, to enter high school quite strong in math.

The order of topics

My seventh grade workbook (contact Jamie York Press for ordering) allows the students to practice their skills with most of the topics listed here, with a few exceptions (e.g., puzzle problems). The order of the units in my workbook is:

1. Arithmetic review
2. Measurement
3. Ratios Part I
4. Percents
5. Ratios Part II
6. Rates
7. Geometry
8. Square Root Algorithm (optional)
9. Algebra (for the *algebra* main lesson)

Arithmetic

Review Sixth Grade

- Especially review fractions, decimals, and division (**see 6th grade Arithmetic**).
- Integrate review into new material, as feasible.

The World of Numbers

Math Tricks

- Review *sixth grade math tricks* (**see Appendix D**).
- Do the *seventh grade math tricks* (**see Appendix D**). Introduce perhaps one new trick each week, and work on practicing new ones with old ones during mental arithmetic. (**See Introduction,** *Mental arithmetic.*)

Divisibility Rules

- Review sixth grade *Divisibility Rules*, and then do these as well**:**
  - A number is evenly divisible by 6 only if it is divisible by both 2 and 3.
    Example: 577,368 is evenly divisible by 6 because it is divisible by both 2 and 3 .
  - A number is evenly divisible by 8 only if the last 3 digits are divisible by 8. This is because it will evenly divide into any number of thousands.
    Example: 8,736,104 is *not* evenly divisible by 8 because the last three digits aren't divisible by 8.
  - A number is evenly divisible by 12 only if it is divisible by both 4 and 3.
    Example: 57,481,932 is evenly divisible by 12 because it is divisible by both 4 and 3.
  - A number is evenly divisible by 11 only if the difference of the sums of every other digit is evenly divisible by 11.

    Example: With 6,273,905, we get one sum by adding the digits 6, 7, 9, and 5 to get 27. The other sum comes from adding the digits 2, 3, and 0, which gives 5. The difference of the two sums is 27−5, which is 22. And since 22 is evenly divisible by 11, then we can say that the original number 6273905 is also evenly divisible by 11.

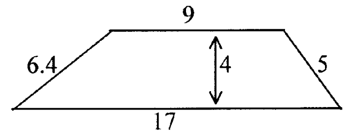
    Example: With 378,543 both sums are equal to 15, making the difference equal to zero. Since zero is evenly divisible by 11, then we can say that 378543 is also evenly divisible by 11.

    Example: With 68,479, the two sums are 19 and 15, which have a difference of 4. Therefore, we conclude that 68479 is not evenly divisible by 11.

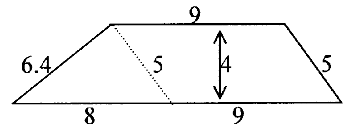
Area of a Trapezoid

- The formula for calculating the area of a trapezoid $A = \frac{1}{2}H(B_1 + B_2)$ is given in math textbooks, but I don't give it to the students. As a challenge problem, I often ask a student to come up with the formula.
- Have the students find the area of any trapezoid by dividing it into a triangle and a parallelogram (or a rectangle), or into two triangles.

  Example: Find the area of the trapezoid shown on the right. Assume all measurements are given in meters.



  Solution: We first divide the trapezoid by drawing a line parallel to one side from one of the obtuse angles, as shown in the drawing at the right. We then calculate the area of the parallelogram as its base (9) times its height, which is 4 (not 5!), resulting in an area of $36m^2$. The triangle also has a height of 4, and its base is 8, so its area is $\frac{1}{2}(8)(4)$, which is $16m^2$. The whole trapezoid, therefore, has an area of $36+16 = \underline{52m^2}$.



Heron's Formula for the Area of a Triangle

Area ΔABC $= \sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}$, where $s = \frac{1}{2}(a + b + c)$ is the semi-perimeter.

- This formula is attributed to the Greek, Heron (fl.≈75A.D.), but it may have been Archimedes that came up with it first.
- Heron's amazing proof of this formula is, for me, the climax of the tenth grade year of studying geometry.
- Before seeing this formula, the students should first be able to calculate the areas of non-right triangles where the base and height are given. (**See 7th grade Geometry,** *Area.*)
- The beauty of this little-known formula is that you don't need to know the height of the triangle. Without this formula, you would have to use trigonometry (studied in high school) to calculate the height, and it would be more complicated.

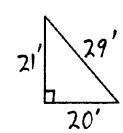
  Example: Find the area of the triangle that has sides of length 5m, 6m, and 7m.

  Solution: The perimeter is 18m, so the semi-perimeter is 9m. Putting all the numbers into the formula, we get: Area $= \sqrt{9(9\text{-}5)(9\text{-}6)(9\text{-}7)}$, which is $\sqrt{9\cdot4\cdot3\cdot2}$, and becomes $\sqrt{216}$. Using the square root algorithm, we get an area of $\underline{14.70m^2}$ (rounded).

Calculating the Area of Four Types of Triangles

- *A right triangle.* We are given the base and the height, so finding the area is easy.

  Example: With the triangle here, the area is: $A=\frac{1}{2}\cdot B\cdot H \rightarrow A=\frac{1}{2}\cdot20\cdot21 \rightarrow A=\underline{210\ ft^2}$



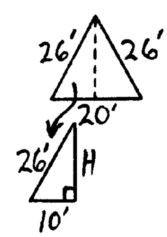
- *An isosceles triangle.* Here, we can use the Pythagorean Theorem in order to calculate the height. We then use this height in order to calculate the area.

  Example: We start with a triangle with one side 20' long and two sides 26' long. To find the height, we cut the triangle in half, which makes a right triangle with sides 26', 10', and H, which is the height of the original triangle. Using the *leg formula* we get:

  $H^2 = 26^2 - 10^2 \quad\rightarrow\quad H^2 = 676 - 100 \quad\rightarrow\quad H^2 = 576 \quad\rightarrow\quad H = 24$

  (We also could have determined H more quickly by using Pythagorean triples.)
  Now we know that the height of the original triangle is 24. So the area is:

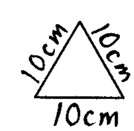
  $A = \frac{1}{2}\cdot B\cdot H \quad\rightarrow\quad A = \frac{1}{2}\cdot20\cdot24 \quad\rightarrow\quad A = \underline{240\ ft^2}$



- *An equilateral triangle.* In this case, we could use the same method as described above for the isosceles triangle, but Heron's formula is generally easier.

  Example: With an equilateral triangle that has all sides equal to 10cm, the perimeter is 30cm, so the semi-perimeter (S) is 15. The area of the triangle is then:

  Area $= \sqrt{15\cdot(15\text{-}10)\cdot(15\text{-}10)\cdot(15\text{-}10)} \rightarrow \sqrt{15\cdot5\cdot5\cdot5} \rightarrow \sqrt{3\cdot5^2\cdot5^2} \rightarrow 5\cdot5\cdot\sqrt{3} \rightarrow 25\cdot(1.73) \approx \underline{43.25\ cm^2}$



- *A scalene triangle* (each side is different). In this case, we must use Heron's formula.

  Example: Using Heron's formula with the triangle here, the perimeter is 72', so the semi-perimeter (S) is half of 72, which is 36. The area of the triangle is then:

  Area $= \sqrt{36\cdot(36\text{-}28)\cdot(36\text{-}24)\cdot(36\text{-}20)} \rightarrow \sqrt{36\cdot8\cdot12\cdot16} \rightarrow \sqrt{55296} \approx \underline{235.1\ ft^2}$