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Seventh Grade Puzzles

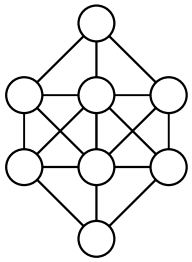
Note: Some of the puzzles in this section could be solved by using algebra. The intention, however, is to solve them without the use of algebra. In this way, the process is likely to be less mechanical and will allow the students to enter more fully into a pure problem-solving experience.

71. Building Chairs

If 3 boys can build 3 chairs in 3 days, how long (at that same rate) does it take 12 boys to build 12 chairs?

72. Connected Circles

Put the numbers 1 through 8 into the circles such that no two consecutive numbers are connected. For example, if we choose to put 3 into the top-most circle, then we cannot put 2 or 4 into any of the three circles just below it.



73. Hand Washing

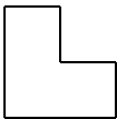
Hank's mother tells him that she will pay him $60 per hour to wash his hands. How much money does he get if it takes him 6 seconds to wash his hands?

74. A Special Number

An eleven-digit number is such that the first digit is 4, the last digit is 7, and the sum of any three consecutive digits is 14. Find the number.

75. Four Congruent Pieces

The figure shown here is as it appears to be – a square with a quarter of it missing. How can it be cut into four congruent pieces?



76. A Generous King

There are 12 people standing in line to receive their gift from the king. The king gives the first person one gold coin and the second person 2 gold coins. The third person has 3 times as much as the second; the fourth person has 4 times as much as the third; the fifth person has 5 times as much as the fourth, and so on. How many gold coins does the last (twelfth) person get?

Games

212. I've Got Your Number!

Grade Level: 5-9
Number of Players: 10+
Math Required: Multiplication Facts
Time to Learn: 10 Minutes
Time to Play: 15 Minutes
Target Situation: Whole Class Activity

This is a great game for practicing factoring and multiplication. Though it is competitive, students can continue to play even after they are eliminated. The game has a clever mechanic whereby students have to do repeated calculations to win, but they are never put on the spot in front of their classmates. It can work for a class of almost any size.

Rules

Have the students make a circle with their chairs and give the following instructions:

- "Stand next to your chairs."
- "Pick a number from 1 to 9; keep it to yourself."
- "You will hear me call different numbers. If your number is a factor of the number I call, step on top of your chair. Once you are there, if your number is a factor of the next number I call, step down. Again, go up and down ONLY if your number is a factor of the number I call. For example, if you picked 2, and I call 13, you do not move, 14, you go up, 16, you go down, etc."
- "After I have called 5 numbers, you can guess which number any of your classmates has picked, if you are wrong, you have to sit down, and continue guessing from your chair. If you are right, that classmate has to sit down and is out of the game."
- "We will continue to play until there are only 4 players left."