## One-Pile Nim

Grade Level: 5-12
Number of Players: 2
Math Required: Addition/Subtraction, Pattern recognition
Time to Learn: 5 Minutes
Time to Play: 2 Minutes

## Target Situation: Small Group

Nim is the simplest of games ${ }^{1}$ and one of the most elegant. It can be played with pencil and paper, or with a chalkboard and eraser, or with a group of similar objects, such as gems. The game has an unbeatable winning strategy.

## Rules

- Game Set-Up. Place a pile of gems (between 7 and 20) on a desk. Instead of gems, you can mark dots on a chalkboard.
- Players alternate turns. With each turn, a player must remove either one or two gems from the pile. Whoever takes the last gem wins.
- Beat the teacher! For older students (seventh grade and above) the real objective should be to figure out the winning strategy. It can work nicely to divide the class into groups of three or four students. Their goal is to figure out the unbeatable strategy, and when a group of students is fully confident (each group only gets one chance!), they can challenge the teacher. The teacher gets to choose the number of gems to begin the game. The students then choose whether they would like to go first, or have the teacher go first. If the students execute the unbeatable strategy without any mistake, then they should win every time. But if they make a mistake, they will lose (presuming that the teacher doesn't make a mistake).
- A variation. Play the same way except that the player to remove the last gem loses.
- The unbeatable strategy for "winner takes the last gem". Fairly quickly, students realize that if they leave their opponent with 3 gems, then they will win. They will also win if they leave their opponent with 6,9 , or 12 gems - any multiple of three - assuming, of course, that no mistakes are made. Lastly, we need to know who should go first. If given the choice, a player should have their opponent go first if the beginning number of gems is a multiple of 3 . Otherwise (if the beginning number of gems is not a multiple of 3 ), the player should elect to go first and then leave the opponent with a multiple of 3 .
- The unbeatable strategy for "loser takes the last gem".

The strategy is nearly the same as above. However, in this case, we realize that we will win if we leave our opponent with $1,4,7,10$, etc., gems. In other words, our goal is to leave our opponent with one more than a multiple of 3 .

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## Two-Pile Nim

Grade Level: 5-12

## Number of Players: 2

Math Required: Addition/Subtraction, Pattern recognition
Time to Learn: 5 Minutes
Time to Play: 2 Minutes
Target Situation: Small Group
This variation of Nim is the same as the above version except that we start with two piles of gems, and with each turn a player may remove as many gems as desired from only one pile. This game also has an unbeatable winning strategy, and for older students the objective is to determine this strategy. Once again, it is good to play the game first with the rule that the player who removes the last gem wins, and then, after mastering that, play with the rule that he who removes the last gem loses.

- The unbeatable strategy. Surprisingly, whether the rule states that the last player to remove a gem wins or loses, the strategy is essentially the same. Either way, if we can leave our opponent with two gems in each pile, we will win. If we begin with an equal number in each pile, then we want our opponent to go first. Otherwise, we will go first and leave our opponent with an equal number in each pile until it gets down to two gems in each pile.


[^0]:    ${ }^{1}$ There are many variations of Nim. They all center around the idea of taking pieces from one or more stacks. This variation is known as Marienbad because it is featured in the artistic film Last Year at Marienbad.

