Lesson Plans for Astronomy

12th Grade Main Lesson –updated Jan 2016

Overview

Astronomy has shaped and guided the development of math and science perhaps more than any other field. And yet, no other topic has become so neglected. Until quite recently, people looked up into the heavens with awe and a sense of security. The heavens provided guidance and wisdom to countless generations of cultures, and at the same time presented itself as a great mysterious puzzle. Our goal together as a class is to become re-connected with the heavens and begin to comprehend some of the mysteries that they present to us.

Notes for the Teacher

- The lesson plans here are based on a 3½ week block.
- <u>Stargazing</u>
  - This block should be scheduled in the winter because it is better for stargazing.
  - I try to take the class stargazing once in the evening and once in the early morning.
  - Stargazing is an important component of the block. The below lesson plans don't include the two stargazing sessions, because it is impossible to say on which day they will take place.
  - Stargazing needs to be scheduled when the moon is close to new, or when it isn't up.
  - After the morning stargazing session, there is no more main lesson.
  - I start each stargazing session by having the students look up into the sky in silence for 10 minutes.
  - During the stargazing sessions, I try to point out the following:
    - *Circumpolar Constellations and Stars*: The Big Dipper, The Little Dipper, Cygnus, Draco, Cassiopeia, Polaris, Vega, Deneb
    - *Winter Constellations and Stars*: Orion, Taurus, Gemini, Auriga, Plaiades, Aries, Leo, the Winter Hexagon, Sirius, Betelgeuse, Rigel, Aldebaran, Procyon, Capella
    - *Other Constellations and Stars* (when possible): Arcturus, Virgo, Spica, Pegasus, Pisces, the Summer Triangle (Vega, Deneb, Altair), Aquarius
    - *Celestial Circles*: The Celestial Equator, The Celestial Meridian, The Milky Way, The Ecliptic (for the second stargazing session),
    - The rotation of the celestial sphere.
  - The lesson after the first stargazing session, I hand out blank star maps and have the students fill in (with me helping a lot!) the stars and constellations that we saw the day before.
- <u>Planetarium visits</u>
  - My email to the presenter for planetarium visit #1 (in the first week of the course):
    - First realize that these 12th grade students have only had a few days of their astronomy course. They know very little about astronomy and have many misconceptions. By the time we arrive at the planetarium, we hopefully will have had one night out stargazing.
    - I am beginning with observational astronomy. So I'd like to keep things as we can see it from the earth, but perhaps accelerating time as needed. Later in the course we will consider the view of the solar system as seen from "above" by an imaginary spaceman.
    - I would like you to focus on the major constellations, perhaps showing them the sky (as seen from Boulder) and then asking them to point to a given constellation, waiting a few seconds and then showing the constellation (as an outline). Here is a list of the constellations that I expect them to locate for their test (at the end of the course): Big Dipper, Little Dipper, Cassiopeia, Draco, Auriga , Summer Triangle, Orion, Gemini, Taurus, Pleiades, Aries, Pegasus, Leo, Canis Major (and Sirius). If you could give them a "practice quiz" on these constellations, that would be great!
    - Show how the constellations move over the course of the day, including the "never-setting" (circumpolar) stars. <u>**Very important**</u>: show how the daily movement of the stars changes depending upon where you are on the earth (e.g., Boulder, North Pole, equator, Australia, etc.)
  - My email to the presenter for planetarium visit #2 (in the last week of the course) includes this list:
    - Show how the daily path of the sun through the sky varies through the course of the year, thereby showing the constellations of the zodiac. Compare the ecliptic and the celestial equator. Show how the ecliptic moves over the course of the day as seen from Boulder.
    - *Retrograde Motion*. Show the movements of the various (five visible) planets over the course of months or years. Just show this, but don't give an explanation - I want them to think about it before I give any explanation.
    - The phases of the moon. Lunar and solar eclipses

Day #1

Introduction

- Two rough-sketch drawings:
  - You are walking on a beach that runs perfectly east-west in the beginning of July. Draw a quick 1-minute sketch of this sunset which includes both the shoreline and the horizon.
  - Recently, I saw a thin crescent moon in the sky just before sunrise. Draw this scene including the moon and an indication of where the sun is about to rise on the horizon.
- How is it possible that our society, which considers itself to be so scientific, can be so ignorant of how the heavens work?
- We are completely disconnected from nature and, in particular, the heavens. It hasn't always been this way. Everyone knew, by looking into the sky, how the sun and the moon moved. They knew every constellation and the many stories that went with it. Grandparents and parents told their children the stories of the stars. It was their entertainment. And they looked to the stars for guidance.

Great Circles on the Earth

- Define great circle: a circle on a sphere which goes "straight" around the sphere, and is the largest possible circle that can be drawn on the sphere. The equator of the earth is an example of a great circle.
- Emphasize first that we are studying two perspectives: Ours from earth, and from a spaceman looking down at the earth.
- From where we stand on Earth, we can imagine three circles:
  (1) our "extended" horizon is our "normal" horizon, that has been pulled downward, out of sight, until it becomes a great circle.
  (2) a meridian circle (running north-south under our feet), and
  (3) a zenith circle (running east-west under our feet). (Note: the zenith is normally referred to only as the point on the celestial sphere directly overhead. I am inventing the term "zenith circle" for reasons that appear clear later.)

Go outside

- Show the three great circles.
- Ask everyone to point to Polaris.
- Ask everyone to run for 50 yards due east.
- Now have the class spread out in the field, but you (the teacher) are in the center of the group. Imagine that you (the teacher) are the exact north pole. Now have everyone run due east for 50 yards.

Longitude & Latitude.

- Show Longitude and Latitude on a globe.
- Boulder is 40°N, 105°W
- Latitude lines are not straight.
- Group work questions:
  - How are longitude lines and latitude lines different from each other?
  - Two planes start out by flying exactly due east. One then flies straight, and the other flies due east. Describe the difference of their flight path.
  - A plane flies from New York to Hanoi, Vienam (which is 12 hours ahead of NY). Which way should the plane take off? (Ans: North)

Hanging Questions:

- If you were relocated to a place on the earth, how could we use the stars to determine your latitude?
- How can we know that the sun is much further from the earth than is the moon?
- How can we know that the earth orbits the sun and not the other way around?

Day #2

<u>The Earth's axis and the Equator</u>

- *The Fourth Great Circle.* The equator is a great circle that is always perpendicular to earth's axis.
- Build up to an understanding that you can determine your latitude by measuring the angle that the North Star is above the horizon. (Adding some angles to the below drawing may help.)
- If I am close to the North Pole, my latitude is high (close to 90°), Polaris (the north star) is high in the sky, and the equator is low.
- If I am close to the equator, my latitude is low (close to 0°), Polaris is low, and the equator is high.

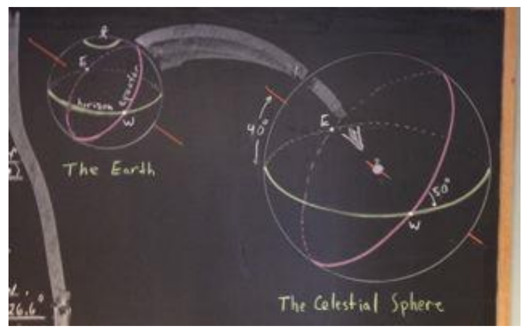
Do Discovery Sheet #1.

- With Discovery Sheet #1, we saw that the zenith circle went from us in an east-west direction. How is it then possible that it ends up heading south to cross the equator? Ans: heading east is not straight.

<u>Homework</u>

- Go outside after dark and locate Polaris. Point one arm at Polaris and the other arm horizontally to the north. What is the angle between your arms?

Day #3



<u>Review</u>

- What did we learn from Discovery Sheet #1? The horizon, zenith, and meridian are all great circles on the earth. They are all at right angles to each other and divide the earth into 8 equal sections.

<u>The Celestial Sphere</u>

- Take all four Great Circles on the Earth and now extend them up onto the Celestial Sphere.
- *The Celestial Meridian*. The sun crosses the celestial meridian at midday, which is the time midway between sunrise and sunset (usually not exactly noon).
- *The Celestial Horizon*
    - Our *normal horizon* is where we see the land (or sea) meet the sky. We can imagine it to be a circle drawn onto the land as far as we can see.
    - Our *extended horizon* is when our horizon has been pulled downward until it becomes a great circle.
    - Our *celestial horizon* is when the extended horizon (or normal horizon) is expanded out onto the celestial sphere. *From our perspective on earth, our normal horizon appears the same (or on top of) our celestial horizon.*
- *The Zenith Circle*. The *zenith* is the point directly overhead on the celestial sphere at any moment in time. The zenith circle goes directly over our heads, from east to west.
- *The Celestial Equator*.
    - The *celestial equator* is the earth's equator projected out onto the celestial sphere. The celestial equator is a great circle that is always perpendicular to earth's axis.
    - While the above three circles appear to be in the same place no matter where we are on earth, the celestial equator is different.
    - It is critically important to be able to "draw" the celestial equator in the sky. *We can easily imagine the celestial equator by tilting the zenith circle (toward the south if we are in the northern hemisphere) by an angle equal to our latitude.*
    - Wherever you are on the earth, the sun travels exactly along the celestial equator on the first day of spring and on the first day of fall.
- From any place on earth, none of the above four circles *ever* appear to move.

<u>Hanging Questions</u>: From the perspective of a spaceman…

- how can we know which way the earth spins (clockwise or counterclockwise) on its axis?
- how can we know which way the earth travels (clockwise or counterclockwise) in its orbit around the sun?

Day #4

- Review
  - Axis and equator are always at right angles to each other.
  - The horizon and zenith are always at right angles to each other.
  - From the perspective of any location on earth, the four great circles never appear to move.
  - How do the four great circles appear to move if you travel from a point in northern Canada and head straight south to Mexico?
- Key concept
  - Carefully build up to an understanding that **the sun's daily path through the sky is always parallel to the celestial equator**.
- Demo explaining the seasons
  - Place a lamp on a desk and then move a globe in a circle around the lamp from about 8 feet away. Stop when it is summer in the northern hemisphere.
  - Question#1: at what angle does the sun appear to be in the sky at midday where we live? (From our location on the globe, have one stick pointing directly at the zenith and one stick pointing at the sun.)
  - Question#2: at sunrise where does the sun appear to be in the sky? (From our location on the globe, have one stick pointing directly east and one stick pointing at the sun.)
  - Question#3: as the earth spins, what region is always in daylight? Never in daylight?
  - Repeat this for when the globe is at the equinoxes and at winter solstice.

Start Discovery Sheet #2

Hanging Question: Why can we not see Orion in the summer (in Boulder)?

Day #5

Review

- From our perspective here on earth, it seems that we are on a flat plane looking up at the celestial sphere. On a clear night without obstruction, we can see exactly half of the stars at any given moment.

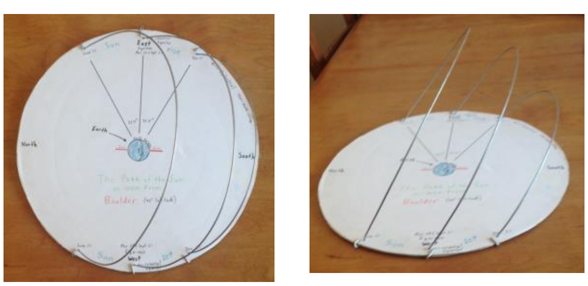
Three ways to view/draw the Earth from "above". Each one is valid!

- With the horizon horizontal (like the drawing on the previous page).
- With the equator horizontal (and the axis vertical).
- With the orbital plane horizontal. This is how most globes are made. (We will learn later that this is the same as the ecliptic.)

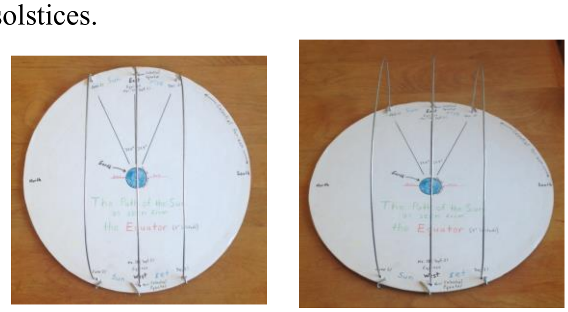
The "correct" way to imagine the celestial sphere

- The celestial sphere has all of the stars on it. The earth is at the center, with the sun, moon, and planets close by. *The sun, moon and planets are not on the sphere, but appear to be so.*

Show my models (shown below) that demonstrate the path of the sun on the first day of each season comparing Boulder and the equator. (I made these from particle-board and wires.) The middle wire shows the sun's path on the equinoxes. The outer wires show the solstices.



Model showing the Sun's Path in Boulder



Model showing the Sun's Path at the Equator

Finish Discovery Sheet #2

Day #6

Do Discovery Sheet #3.

- After the students have worked on Discovery Sheet #3, write these key thoughts on the board:
    - Further north than the tropic of Cancer (23.4°N), the sun at noon is in the south every day of the year, and it travels through the sky from left to right.
    - Further south than the tropic of Capricorn (23.4°S), the sun at noon is in the north every day of the year, and it travels through the sky from right to left.
    - In the tropics (between 23.4°N and 23.4°S), the sun at noon is sometimes in the north and sometimes in the south. Sometimes it travels through the sky from right to left, and sometimes from left to right.

Day #7

- Handout Summary Sheet #1. Students should look over it groups to be sure that they understand everything.
- <u>Key Demo for showing the Ecliptic</u>
    - In a large space (preferably a gym), have a model of a sun at the center of the room.
    - With a globe, have the earth rotate around the sun – closer to the sun than the walls of the room.
    - Stop when it is the summer solstice and ask: *If you are looking at the sun from the earth, and you could magically turn off the sun, where would the stars be located on the celestial sphere that are behind the sun?* Mark this spot on the wall of the room with two pieces of paper labeled "Taurus" and "Gemini", with Gemini on the left.
    - Continue. Stop at the autumnal equinox (Virgo), Winter solstice (Sagittarius), Vernal equinox (Pisces).
    - Fill in the rest of the zodiacs by using the zodiac table.
    - <u>**This circle on the celestial sphere is called the ecliptic**</u>. The ecliptic and the celestial equator are both great circles on the celestial sphere. They intersect at two places: in the constellations of Virgo (which is where the sun is on the autumnal equinox) and Pieces (where the sun is on the vernal equinox).
    - Now locate Orion (which is on the celestial sphere, and high in the sky in the winter). It is directly underneath Taurus and Gemini. <u>Key thought</u>: Because Orion is close to Taurus and Gemini, which is where the sun is in the summer, Orion is not visible in the summer. It is most visible in the winter.
    - Discuss why a certain stars are not visible at certain times of the year. Others are always visible.
    - Important: Notice that from the perspective of someone on earth, the sun moves from right to left against the background of the stars. If the earth's orbit was instead clockwise, then it would move from left to right.

Day #8

Catch-up day!

<u>Key/Hanging Questions</u>:

- Why can't you see planets in the northern sky?
- Given yesterday's demo, where do the Celestial Equator and the Ecliptic intersect?
- Which planets can ever appear (through the use of a telescope) as "full"? As a crescent?
- How long does it take for the sun to rise on the equator? In Boulder? At the North Pole? (See worked out answer at the end of these lesson plans.)

Day #9

Planetarium visit #1!!

Day #10

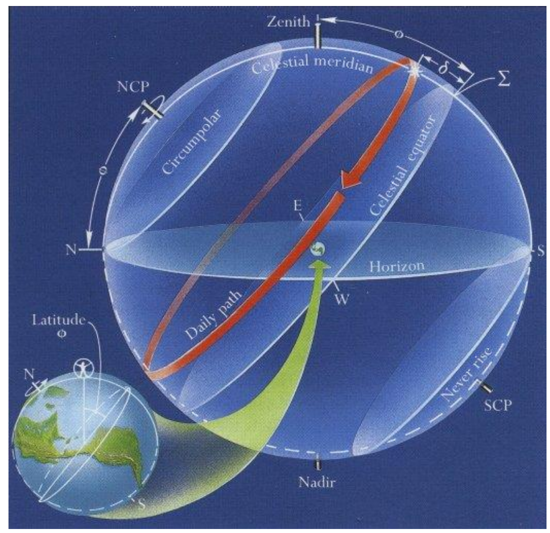
Review

- Review the key ideas from the planetarium
- If we imagine a video, where a photo is taken each day of the year such that the stars are in the same place in the sky, then we can say the following: (1) From one day to the next, the photo was taken 4 minutes early than the previous day; (2) On the days that the photo is taken when the sun is up, the sun appears to be traveling from right to left against the background the (invisible) fixed stars.

Handout Summary Sheet #2. Students should look over it in groups to be sure that they understand everything.

The Stars

- Introduce the idea of "never-setting" and "never-rising" stars. The Stars on the Celestial Sphere fall into three categories:
- The stars that "never set". These are the circumpolar stars, which, in Boulder (40° latitude), comprise about 12% of the stars on the celestial sphere.
- The stars that "never rise". For us in Boulder, these are the stars that circle the southern celestial pole – again, about 12% of the stars on the celestial sphere.
- The stars that rise and set every day. Where we are in Boulder, these stars are about 76% of the stars on the celestial sphere. Therefore, in Boulder, we can see (at some point in the year) about 88% of the stars on the celestial sphere.
- See the drawing at the right, which is for 40° latitude, like Boulder.



How Many Stars are Visible?

- Given that from any place on earth, at any given in the night, we can see half of the celestial sphere…
    - There are 9096 stars on the whole celestial sphere that can be seen with the naked eye. This means that in the best of conditions, approximately 4500 stars are visible at a given moment from a given location.
    - On a clear night in the suburbs, about 450 stars are visible.
    - On a clear night in a large city, about 45 stars are visible.

Do Discovery Sheet #4

- It includes these figures on "Relative Distances":
    - For an observer on the ground (5 ft 7 in) (average eye-level height), the horizon is 2.9 miles away.
    - The "furthest sight distance" one can possibly see on a very clear day (200 miles) is about 70 times further than that.
    - The circumference of the earth (24,880 miles) is about 125 times further than that.
    - The distance to the moon (239,000 miles) is about 10 further than that.
    - The distance to the sun (93,000,000 miles) is about 389 times further than that.
    - The distance from the sun to Neptune (the furthest planet) is about 30 times further than that.
    - The nearest star (Alpha Centauri 4.24 light years away) is about 9,400 times further away than that.
    - The diameter of the Milky Way Galaxy (110,000 light years) is about 25,000 times further than that.
    - Andromeda galaxy (the nearest galaxy 2.25 million light years away) is about 20 times further than that.
    - The diameter of the "observable universe" (50 billion light-years) is about 20,000 times greater than that.

Day #11

Review

- Why do the stars rise 4 minutes earlier each day? (Ans: because we saw in our demo that the sun moves from right to left against the background of the stars – the opposite direction of travel.)
- Discuss the "Wobble" of the ecliptic. There are two places where the celestial equator and the ecliptic intersect: Virgo and Pisces. Every day these two constellations travel along the celestial equator, dragging the wobbling ecliptic along with it.

The Zodiac. Give the "Zodiac Table" handout. The differences between the following:

- *Tropical Zodiac*. The (original) tropical zodiac originates with Babylonian astronomy nearly 3000 years ago. The zodiac signs were assigned according to where the sun was at that time, and divided them evenly into 12 segments starting with Aries on the first day of spring.
- *Sidereal Zodiac*. The (modern) sidereal zodiac is shifted from the tropical zodiac by about 25 because the sun is now in a different position in the zodiac. Due to the procession of the equinox (see below) the zodiac dates move about one day every 70 years.
- *Astronomical Zodiac*. The astronomical zodiac shows where the sun is in each constellation without dividing it equally. Note that there are 13 constellations on the ecliptic, but that Ophiuchus is left out of the zodiac.
- *Precession of the Equinox*. This is the "wobbling" of the earth. It takes about 25,800 years for the earth to make one full gyroscopic precession. This is why the tropical and sidereal zodiac dates are about 25 days out of sync.
- *Astrological Age and Cultural Epoch* (an anthroposophical perspective; for the teacher only). An astrological age corresponds to which zodiac the vernal equinox falls on. It takes approximately 2,150 years ($25{,}800 \div 12$) to move from one astrological age to the next. There is little agreement about when each astrological age begins – for example, the Age of Aquarius is said to begin somewhere between 1400 and 3600 AD. With Steiner, each cultural age ("epoch") begins 1199 years after the vernal equinox enters a given zodiac.
    - *Aries*. Steiner states that the Greco-Roman cultural epoch (which corresponds to the "Age of Aries") was from 747 BC until 1413/14 AD. The astronomical age of Aries began in 1946 BC, when the vernal point entered Aries.
    - *Pisces*. The cultural age of Pisces (Steiner's 5th "cultural epoch") is from 1413 to 3573 AD. The astronomical age of Pisces is from 215 AD to 2375 AD.
    - *Aquarius*. The cultural age of Aquarius (Steiner's 6th "cultural epoch") is 3573 to 5733 AD. The astronomical age of Aquarius is from 2375 to 4535 AD.

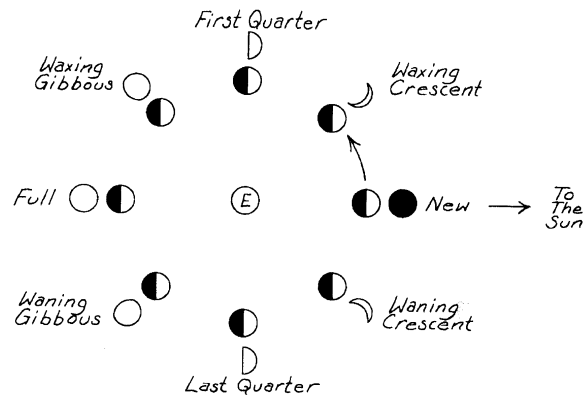
Do Discovery Sheet #5. This requires a celestial globe and a celestial umbrella.

Day #12

Review
- Everything from Discovery Sheet #5.

Handout Summary Sheet #3. Students should look over it groups to be sure that they understand everything.

Phases of the Moon
- Show the drawing at the right on the board.
- Note for the teacher: Students have many misconceptions about what causes the moon's phases.



Day #13

Do Discovery Sheet #6 (On the Phases of the Moon)

Hanging Question: We can now picture the daily and monthly path of the moon in our sky? If you were on the moon, what does the daily and monthly path of the earth look like?

Day #14

Catch up day

Day #15

Planetarium visit #2!!

Day #16

Review
- Visit to the Planetarium.

Handout Summary Sheet #4. Students should look over it groups to be sure that they understand everything.

- Handout Sheet titled: "Orbits and Rotations of Planets"

Do Discovery Sheet #7 (On the Planets)
Demo
- Imagine a planet that rotates (spins) 8 times per year. Assume that's its orbit is counter-clockwise. How many "days" does it have if it rotates counter-clockwise? Clockwise? (Ans: 7 and 9)

Review
- Handout Summary Sheet #3. Students should look over it groups to be sure that they understand everything.

Day #17

Possible discussion and essay topic: *Are we insignificant?*

If you would like to get a head start on your essays for next week, then you can consider the below topic as an essay. (No double essay, please.)

When studying astronomy, we may encounter the question: Are we the only planet in the universe that has (intelligent) life? The typical argument is quite convincing. There are countless stars and planets in the universe. What is the probability that no other planet has life? Even if only 0.0000001% of all planets has life, given the total number of planets in the universe, there must be many planets, other than just others, that support life.

I'd like to give my opinion. While I have no viable argument against the conclusion that many other planets have intelligent life, I am not so convinced of the above argument. For me it isn't simply a question of probabilities. It is a philosophical and spiritual question. Is the above scientific argument essentially an atheist argument? Are we assuming that there is no God? Are we assuming that the mere existence of life is simply random chance and not a miracle? Are we really not special at all – in fact, quite insignificant? How can I know?

This reminded me of a popular quote from the famous Cornell astronomer, Carl Sagan (1934-1996):

> *"Who are we? We find that we live on an insignificant planet of a humdrum star lost in a galaxy tucked away in some forgotten corner of a universe in which there are far more galaxies than people."*

It is interesting to note that Sagan was not an atheist, but rather, was agnostic (someone who neither believes nor disbelieves in the existence of a deity or deities). In fact, Sagan once said:

> "*Science is not only compatible with spirituality; it is a profound source of spirituality…When we grasp the intricacy, beauty, and subtlety of life, then that soaring feeling, that sense of elation and humility combined, is surely spiritual.*"

Here is a link to an interview of a Vatican astronomer, who talks about God and astronomy.

http://www.freep.com/story/news/local/michigan/2015/09/22/pope-francis-guy-consolmagno-vatican-astronomer/72572258/

He says that we do astronomy because it satisfies a "hunger in our souls", and that "science is a way of getting to know God the creator". (This is the only time you may use the Internet for this course.)

What do you think?

Johannes Kepler's Laws of Planetary Motion

- 1596: In 1596 he published the book, *Mysterium Cosmographicum* in which he stated his hypothesis on the movement of the planets. It was based on the idea that the Platonic solids could be nested inside one another, and thereby defined the spacing of the planets. In short, the order was: sphere (Mercury), octahedron, sphere (Venus), icosahedron, sphere (Earth), dodecahedron, sphere (Mars), tetrahedron, sphere (Jupiter), cube, sphere (Saturn). After obtaining Tycho Brahe, he realized his theory was wrong (but very close!).
- (1610 & 1619): Publishes his three laws of planetary motion:

1. The orbits of every planets is an ellipse with the sun at one of the focal points.
2. The line joining any planet to the sun sweeps out equal area in equal time.
3. The squares of the revolutionary periods of any two planets is directly proportional to the cubes of their mean distance from the sun.

Example: If one planet is 4 times as far from the sun as another, then it takes that planet 8 times as long to go around the sun. This is because $8^2 = 4^3$.

Example: since Mars is 1.52 times as far from the sun as the earth is, this means that it's period (P) can be found from: $P^2:(1\text{ year})^2 = (1.52)^3:(1\text{AU})^3$, yielding an orbit of ≈1.87 years.

Example: Neptune's period is about 13.9 times as long as Jupiter's. Therefore, Neptune must be $13.9^{2/3} \approx 5.781$ times as far from the sun as is Jupiter.

Key conversation:

- The celestial sphere is a mental construct. In reality, the stars are not on a sphere. Are we doing "real science" when we use the celestial sphere?

Day #18

- <u>Each epoch in history</u> can be thought of as having a dominant way of thinking or being.
  - In the world of math and science, it was the Greeks that began a new paradigm. This paradigm is exemplified by Euclid and culminated in science with Newton.
  - A new paradigm is now beginning. Some people have made the shift; most have not.
  - Example using the earth and sun:
    - Previous to the Greeks, everyone "knew" that the sun went around the earth. Nobody wondered about it. The erratic movement of the planets was noticed, but nobody felt a need to explain it.
    - The Greeks then tried to provide an explanation for the movement of the planets. These attempts continue for over 2000 years, until Kepler solves the mystery. But still, for quite a while, most people are stuck thinking in the old paradigm – not questioning what they see, and not needing a logical explanation.
    - The new epoch requires a new paradigm for thinking in math and science. This new paradigm includes relativistic, holistic, "paradoxical' thinking (e.g., parallel line mystery, relativity, etc.)
      - It is possible (as a thought-experiment) to use a video camera to show the earth at the center of the screen, and the sun rotating around the earth. Both the geocentric and the heliocentric models are "valid". Motion is relative. The heliocentric model is still superior because it is simpler. The geocentric model is possible, but the movement of the planets would be very complicated.

<u>Extra Challenge Questions</u>:

- *How long does it take for the sun to rise at the North Pole*?
  $\theta = 23.4 \sin(\frac{2\pi}{365} D)$ where D = # days past spring equinox, and θ is the elevation of the sun in the sky.
  $\frac{d\theta}{dD} = 23.4 \cdot \frac{2\pi}{365} \cdot \cos(\frac{2\pi}{365} D)$ This tells us the rate at which the sun is rising (or lowering) at any day of the year. On Spring equinox, when the sun begins to rise at the North Pole, D = 0. This tells us that the sun is gaining elevation on that day at a rate of $23.4 \cdot \frac{2\pi}{365} \approx 0.4045$ degrees per day. Since the sun appears to be about 0.5 degrees in the sky, we can say that the sunrise will take $0.5 \div 0.4045 \approx 1.24$ days to rise, which is about 29 ¾ hours
- *When is Jupiter least illuminated* (from our perspective on earth)? (Note: that it is always almost full.)
  If J, E, S represent the positions of Jupiter, Earth, and Sun at a given time, then we can create a triangle where $\alpha = \angle EJS$ and $\theta = \angle JES$. Let ES = 1, therefore JS ≈ 5.2. using the law of signs gives us:
  $\theta = \sin^{-1}\left(\frac{\sin\alpha}{5.2}\right)$ With thinking, we can realize that Jupiter appears full when θ is zero, and it is least illuminated when θ is as big as possible. Clearly, this happens when $\frac{\sin\alpha}{5.2}$ is as large as possible, which is when α = 90°, which we may have suspected all along) and θ ≈ 11.1° (which is how much, out of 180° that Jupiter isn't illuminated).
- *When is Venus the brightest in the sky?*
  I have some ideas of how to solve this, but not the time quite yet.