The Mathematics of the Game "Set"

- There are a total of 1080 unique sets.
  - 108 of these sets (10% of all sets) have 3 attributes that match.
    - # ways to choose 3 attributes and their types = $\frac{12 \cdot 9 \cdot 6}{3!}$ = 108.
    - # ways to choose 0 non-matching attributes = 1.
    - Total # of ways to have this kind of set = 108•1 = 108.
  - 324 of these sets (30% of all sets) have 2 attributes that match.
    - # ways to choose 2 attributes and their types = $\frac{12 \cdot 9}{2!}$ = 54.
    - # ways to choose 1 non-matching attribute = 3! = 6.
    - Total # of ways to have this kind of set = 6•54 = 324.
  - 432 of these sets (40% of all sets) have 1 attributes that match.
    - # ways to choose 1 attribute and its type = $\frac{12}{1!}$ = 12.
    - # ways to choose 2 non-matching attributes = $(3!)^2$ = 36.
    - Total # of ways to have this kind of set = 12•36 = 432.
  - 216 of these sets (20% of all sets) have 0 attributes that match.
    - # ways to choose 0 attributes and their types = 1.
    - # ways to choose 3 non-matching attributes = $(3!)^3$ = 216.
    - Total # of ways to have this kind of set = 1•216 = 216.
- The odds against there being no Set in 12 cards when playing a game of Set start off at 30:1 for the first round. Then they quickly fall, and after about the 4th round they are 14:1 and for the next 20 rounds they slowly fall towards 13:1. So for most of the rounds played, the odds are between 14:1 and 13:1. (I have not yet figured out a way to derive the value of 30:1.)
- The odds against there being no Set in 15 cards when playing a game are around 90:1. (Note that the Set instructions manual gives the odds at 2500:1, but that is assuming that 15 random cards are selected, which never happens; 15 cards are only on the table right after 12 cards are known to have no sets.)
- The largest group of cards you can put together without creating a set is 20.
- If 26 Sets are drawn from a full deck (of 81 cards), the remaining 3 cards must also form a Set (i.e., it is impossible to end a game with just 3 cards on the table).
- Around 30% of all games always have a Set among the 12 cards, and thus never need to go to 15 cards.
- Given any two cards, there exists one and only one card which forms a set with those two cards. Therefore, the probability of choosing 3 random cards from a complete deck and getting a set is $^1/_{79}$.
- Since the number of possible combinations of three cards chosen from twelve cards is 12C3 (12C3 = 12!/[(12-3)!3!] = 220), the expected number of sets in a group of 12 cards (first deal) is 1/79 times 12C3, or approximately 2.78. (I'm still not convinced that this logic is sound; 2.78 may be incorrect. The logic for getting 2.78 would definitely be correct if we were selecting a group of three cards, 220 times, each time from a full deck – but this is not what we are doing here. I have also seen 2.4 as the average number of sets in the first deal.)