# A Math Paradox: The Widening Gap Between High School and College Math

**By Joseph Ganem** (American Physical Society;
http://www.aps.org/publications/apsnews/200910/backpage.cfm?renderforprint=1)

We are in the midst of paradox in math education. As more states strive to improve math curricula and raise standardized test scores, more students show up to college unprepared for college-level math. The failure of pre-college math education has profound implications for the future of physics programs in the United States. A recent article in my local paper, the *Baltimore Sun*: “A Failing Grade for Maryland Math,” highlighted this problem that I believe is not unique to Maryland. It prompted me to reflect on the causes.

The newspaper article explained that the math taught in Maryland high schools is deemed insufficient by many colleges. According to the article 49% of high school graduates in Maryland take non-credit remedial math courses in college before they can take math courses for credit. In many cases incoming college students cannot do basic arithmetic even after passing all the high school math tests. The problem appears to be worsening and students are unaware of their lack of math understanding. The article reported that students are actually shocked when they are placed into remedial math.

The article did not shock me. It described my observations exactly. In recent years I’ve witnessed first hand the disconnect between the high school and college math curricula. As a parent of three children with current ages 14, 17, and 20, I’ve done my share of tutoring for middle school and high school math and I know how little understanding is conveyed in those math classes. Ironically much of the problem arises from a blind focus on raising math standards.

For example, the problems assigned to my children have become progressively more difficult through the years to the point of being bizarre. My wife keeps shaking her head at how parents without my level of math expertise assist their children. My eighth-grade daughter asked me one evening how to perform matrix inversions. I teach matrix inversion in my sophomore-level mathematical methods course for physics majors. It is difficult for me to do matrix inversions off the top of my head. I needed to refresh my memory by pulling Boas’ book: *Mathematical Methods in the Physical Sciences* off my shelf. Not exactly eighth grade reading material.

On another night my eighth-grader brought home a word problem that read: If John can complete the same work in 2 hours and that it takes Mary 5 hours to complete, how much time will it take to complete the work if John and Mary work together? That’s an easy problem if you know about rate equations. Add the reciprocals of 2 and 5 and reciprocate back to get the total time. However it took me a lot of thought to arrive at an explanation of my method comprehensible to an eighth-grader.

My other daughter struggled through a high-school trigonometry course filled with problems that I might assign to my upper-class physics majors. I certainly wouldn’t assign problems at such a high level to college freshmen. I kept asking her how she was taught to do the problems. I wondered if the teacher knew special techniques unknown to me that made solving them much easier. Alas no such techniques ever materialized. The problems were as difficult as I judged. At least I could solve the problems, a feat the teacher couldn’t manage in a number of cases.

For example one problem involved proving a complicated trigonometric identity. My daughter brought it to me saying she had tried but couldn’t find a solution. I saw immediately that the textbook had an error that rendered the problem meaningless. One side of the problem had a combination of trigonometric functions with odd symmetry and for the other side the symmetry was clearly even. I told her it was not an identity and that fact could be proven with a simple numerical substitution on each side. If it is an identity the equality condition must hold for all values of the angle. A single numerical counter example proves that it is not an identity. It only took one try to find a counter example.

The next day she reported to me that the teacher couldn’t solve the problem.

“Did you tell him that it is impossible?” I asked.

“I told him it was not an identity and if he put numbers in he would find that out. He didn’t believe me. He just said ‘We’ll see’.”

The teacher never talked about that problem again. He did teach the class about the symmetry properties of trigonometric functions but evidently he didn’t understand the usefulness of that knowledge.

At the same time I work the summer orientation sessions at Loyola College registering incoming freshmen for classes. Time and again students cannot pass the placement exam for college calculus. Many students cannot pass the exam for pre-calculus and that saddles them with a non-credit remedial math course—the problem described in the newspaper article. Without the ability to take college-level math the choices students have for majors are severely limited. No college-level math course means not majoring in any of the sciences, engineering, computer, business, or social science programs.

A colleague in the engineering department who also works summer orientation complained to me that many students who wanted to major in engineering could not place into calculus. The engineering program is structured so that no calculus means no physics freshman year and no physics means no engineering courses until it's too late to complete the program in four years. For all practical purposes readiness for calculus as an entering freshman determines choice of major and career. The math placement test given to incoming freshmen at orientation has much higher stakes than any test given in high school. But, the placement test has no course grade or teacher evaluation associated with it. No one but the student has any responsibility for or stake in its outcome.

Through the years I've found it discouraging as a faculty member to see so many high aspirations dashed at orientation before classes even begin. I tell students with poor math placement scores to go home, review high school math over the summer and take the test again. But, few take my advice. Most students with poor placement scores switch to majors that do not have significant math requirements.

So if eighth graders are taught math at the level of a college sophomore why are graduating seniors struggling? How can students who have studied college level math for years need remedial math when they finally arrive at college? From my knowledge of both curricula I see three problems.

1. Confusing difficulty with rigor. It appears to me that the creators of the grade school math curricula believe that "rigor" means pushing students to do ever more difficult problems at a younger age. It's like teaching difficult concerti to novice musicians before they master the basics of their instruments. Rigor–defined by the dictionary in the context of mathematics as a "scrupulous or inflexible accuracy"–is best obtained by learning age-appropriate concepts and techniques. Attempting difficult problems without the proper foundation is actually an impediment to developing rigor.
   Rigor is critical to math and science because it allows practitioners to navigate novel problems and still arrive at a correct answer. But if the novel problems are so difficult that a higher authority must always be consulted, rigorous thinking will never develop. The student will see mathematical reasoning as a mysterious process that only experts with advanced degrees consulting books filled with incomprehensible hieroglyphics can fathom. Students need to be challenged but in such a way that they learn independent thinking. Pushing problems that are always beyond their ability to comprehend teaches dependence–the opposite of what is needed to develop rigor.

2. Mistaking process for understanding. Just because a student can perform a technique that solves a difficult problem doesn't mean that he or she understands the problem. This is the problem with teaching eighth-graders techniques such as matrix inversion. The arithmetic steps can be memorized but it will be a long time, if ever, before the concept and motivation for the process is understood. That raises the question of what exactly is being accomplished with such a curricula? Learning techniques without understanding them does no good in preparing students for college. At the college level emphasis is on understanding, not memorization and computational prowess.

3. Teaching concepts that are developmentally inappropriate. Teaching advanced algebra in middle school pushes concepts on students that are beyond normal development at that age. Walking is not taught to six-month olds and reading is not taught to two-year olds because children are not developmentally ready at those ages for those skills. When it comes to math, all teachers dream of arriving at a crystal clear explanation of a concept that will cause an immediate "aha" moment for the student. But those flashes of insight cannot happen until the student is developmentally ready. Because math involves knowledge and understanding of symbolic representations for abstract concepts it is extremely difficult to short cut development.

All three of these problems are the result of the adult obsession with testing and the need to show year-to-year improvement in test scores. Age-appropriate development and understanding of mathematical concepts does not advance at a rate fast enough to please test-obsessed lawmakers. But adults using test scores to reward or punish other adults are doing a disservice to the children they claim to be helping.

It does not matter the exact age that you learned to walk. What matters is that you learned to walk at a developmentally appropriate time. To do my job as a physicist I need to know matrix inversion. It didn't hurt my career that I learned that technique in college rather than in eighth grade. What mattered was that I understood enough about math when I got to college that I could take calculus. Memorizing a long list of advanced techniques to appease test scorers does not constitute an understanding.

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