Re-Imagining Mathematics

by Jamie York

Our Mathematical Crisis

It is well known that the U.S. education system – our teaching of mathematics and science, in particular – is in crisis. Regularly, we hear reports about how poorly U.S. schools fair in studies ranking mathematics education across the globe. Depending upon the study, the U.S. typically ranks rather unimpressively between 28th and 35th in the world[1]. The term "crisis" may give the impression that this is something new, or that if we really did something about it, we could fix it relatively quickly. Neither of these assumptions is true; this is not a recent phenomenon, and there is no easy fix. The problem is both complex and deep and exists not just in our mainstream schools, but in our Waldorf institutions as well. The solution, I contend, begins with a re-definition of the nature and purpose of mathematics. I've reflected on this long and hard, and it has led me to uncover the root cause of the math crisis. And I have a solution.

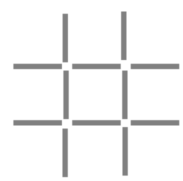
Let me start with a story. Some time ago at a social gathering, I introduced myself to a woman named Beth. I mention in passing that I am a math teacher, and suddenly my new acquaintance is uneasy, as if I just revealed my history as a psychopath. I manage to continue the conversation and discover that Beth is suffering from math trauma, even though she hasn't sat in a math classroom for 20 years. Beth says that in the early grades, she liked math and was good at it. Then, in middle school, math no longer made sense, and math class turned into a form of torture. Beth fell off the math train.

I have met many people over the years with similar stories. I can confidently say that the majority of people in the U.S. had negative experiences with mathematics as students. They then go through their adult lives saying, "I'm not good at math." Why is that? My answer is simple: *They can't say they aren't good at math – they never had any math*! In fact, very few people truly understand what math is.

What is Math?

The following puzzle illustrates my point.

<u>Puzzle #1</u>: The diagram on the right shows 12 sticks in a particular arrangement. Move three sticks, so that you end up with exactly three squares. (Every stick must be part of a square. No two sticks may be placed on top of each other or side-by-side.)
Get out some toothpicks and try it! Continue reading the article once you are done.



What was it like for you doing that puzzle? If you found it hard, did you give up or get determined? This is math! *Struggle is an important part of doing math.*

Blind Procedures

Think of all of those hours you spent sitting in math class as a student. What did you do? Most likely, the teacher introduced some new concept, and then, soon afterwards, showed you how to solve a couple of related problems. You practiced some of these in class, and then many more as homework. After a couple of weeks, you took a test to ensure that you had "learned" the material. Sound familiar? And how did you get your "A" on all of those math tests? You simply remembered how to do each of the problems – right? Did you have to understand what you were doing? Not really. And that's the issue.

For most people, math is a collection of blind procedures used to solve meaningless problems.

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[1] "Asia tops biggest global school rankings", BBC News, 13 May 2015.
http://www.bbc.com/news/business-32608772
"U.S. students improving – slowly – in math and science, but still lagging internationally", Pew Research Center, February 2, 2015.
http://www.pewresearch.org/fact-tank/2015/02/02/u-s-students-improving-slowly-in-math-and-science-but-still-lagging-internationally/

Let's look at this statement more closely. Here are some problems that could typically appear on a math test:

1. Find the sum: $\frac{1}{3} + \frac{3}{10}$
2. Divide: $2516 \div 37$
3. Find the area of a circle that has a radius of 10cm.
4. Solve $5x - 3 = 2x + 21$

For each of the above problems, without remembering the appropriate procedure, a student has no chance of finding the correct answer.

"But doesn't math have to involve procedures?" you might protest. To some degree, the answer is yes. However, there are three pitfalls with procedural math:

- *Blind procedures*. All too often, procedures are taught blindly. Instead, we need to find ways to introduce them so students understand the math behind the concepts. This develops mathematical thinking.
- *Too much*. There is an overemphasis on procedures and "math skills." Math is much more than this.
- *Too young*. We teach too much to students who are too young because of the overemphasis on procedures and skills, the tendency to teach procedures blindly, and the fear that our students will fall behind.

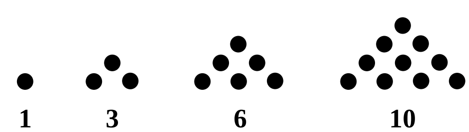
The main theme here– that people don't understand what math is – certainly helps to explain what happens to our students in adolescence. So many procedures that make no sense! After a while, it becomes too much, and they fall off the math train.

Math should be an adventure!

Mainstream math textbooks are often packed with contrived problems to demonstrate that math is really fascinating and important. Parents and teachers confirm that math is terribly dull and meaningless for students when they say things like: "Well, you'll need to use math later in life", or: "Math is a language for science and other subjects that you'll study in college." This is quite sad. There is so much more to math than "learning" a huge list of procedures and skills that you will need in some distant future.

Paul Lockhart, a high-level university mathematician turned elementary school math teacher, wrote an essay in 2002, titled *A Mathematician's Lament*[2], in which he describes the modern mainstream math curriculum as "soul-crushing". He argues that just like poetry, music, painting, and literature, mathematics is an art, and should be taught as such. "Math is *not* a language", he says, "It's an adventure!" What would it be like for our students if math were an adventure? What does a mathematical adventure look like? Here's something for fifth graders:

<ins>Puzzle #2</ins>: The ancient Greeks were interested in patterns. One such sequence of numbers they investigated (shown on the right) is called triangular numbers. Answer the following questions:



a) What are the next ten triangular numbers?

b) What are the square numbers?

c) Is there anything special that you notice about these two kinds of numbers?

Triangular and square numbers are a guaranteed way to engender enthusiasm in the fifth grade classroom. Did you notice that the spacing between the square numbers (1, 4, 9, 16, 25…) is consecutive odd numbers? In other words, going from 1 up to 4, I have to add **<ins>3</ins>**; going from 4 to 9, I have to add **<ins>5</ins>**; going from 9 to 16, I have to add **<ins>7</ins>**, etc. What a wonderful thing it is when the students discover – rather than the teacher telling them – this mathematical law. Even more exciting is when the students discover that any two consecutive triangular numbers always add up to a square number (e.g., 3+6 = **<ins>9</ins>** or 6+10 = **<ins>16</ins>** or 10+15 = **<ins>25</ins>**). That's a mathematical adventure!

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[2] *A Mathematician's Lament*, by Paul Lockhart, 2002. https://www.maa.org/external_archive/devlin/LockhartsLament.pdf.

The Role of Fear

Of course, you may be thinking, “Math puzzles and mathematical adventures sound nice, but I want my kid to be prepared for college.” A certain fear underlies this statement (one I have heard all too often). The unconscious assumption is the existence of a long list of stuff that every student must know in order to succeed in college and life. The term “No Child Left Behind” fills us with fear that our child will lose in the race to get ahead. The consequences are a math curriculum crammed with too many topics, superficial learning, and one blind procedure after another.

Let’s now step back from this place of fear and ask, “What does it mean to be prepared for college?” When I pose this question to parents and teachers, I get agreement that it’s about these three goals: (1) instilling enthusiasm for learning; (2) developing mathematical thinking, and
(3) mastering basic skills[3]

Focusing on these three fundamental goals can completely transform our teaching. Think about it…does the current skills-dominated approach (with 95% of math classroom time spent on skills work) help us to achieve our three goals? Clearly it can’t. How can slogging through endless, meaningless problems be the best route to developing mathematical thinking and instilling enthusiasm for learning (our first two goals)? And furthermore, the current mainstream approach doesn’t even satisfy our third goal (mastering basic skills) – even though students spend far too much time on skills, it doesn’t stick. Don’t get me wrong – I know that mastering basic skills is important. However, if we use our classroom time prudently and we teach so that our students understand the math behind the procedures, then we only need to dedicate about 50% of classroom time to skills – and it will stick better.

Now let’s investigate how we can better develop mathematical thinking.

Developing Mathematical Thinking

Here’s a good puzzle that works nicely for high school students, or you, my adult reader. Be aware that the four questions, below, get increasingly difficult. Try to find your appropriate level of difficulty.

Puzzle #3: Every positive whole number can be said to have a certain number of factors. A factor is simply a number that divides evenly into a given number. For example, the number 50 has 6 factors (1, 2, 5, 10, 25, 50). The questions are then as follows:

a) Give at least one number that has exactly 8 factors.

b) Give at least one number that has exactly 5 factors.

c) Give at least one number that has exactly 13 factors.

d) What is the smallest number that has exactly 45 factors?

This is intended to provide you with a true problem-solving experience. Of course, the first thing to go through a student’s head when encountering such a problem should be, “I have never seen anything like this before. I have no idea what to do.” That’s exactly what problem solving is. Problem solving is an important tool for developing mathematical thinking. It is a tragedy of our mathematics education today that such problem-solving experiences have almost no place in our classrooms.

Now that we have spoken about what it takes to prepare our students for college-level mathematics, I must acknowledge that a truly meaningful education is not just about preparing for college. Rudolf Steiner spoke emphatically about a higher purpose for the teaching of mathematics.

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[3] The list of necessary basic skills is actually quite manageable, if you really think about it.

The Higher Purpose of Teaching Math

- *Math teaches us how to think*. We want our students to be able to think for themselves, think analytically, and we hope that their thinking is heart-felt and imbued with imagination.
- *Character development.* Math teaches students discipline, perseverance, patience, and how to deal with mistakes. For many students, math teaches them how to (work through) struggle.
- *Combating cynicism*. In today's world, our youth can easily come to believe that things are meaningless and without purpose. Cynicism is a pervasive social disease today. By teaching math in the right way, we can show our students that the world is true and filled with awe and wonder.
- *Spiritual and moral development*. Steiner speaks of mathematics as training in sense-free thinking, thus an important part of a student's moral and spiritual development.

  > "The student of mathematics must get rid of all arbitrary thinking and follow purely the demands of thought. In thinking in this way, the laws of the spiritual world flow into him. This regulated thinking leads to the most spiritual truths." [4]

- *Math is human.* Math, like music, drama, painting, and literature, is an art. Through these subjects we learn what it means to be human.

This more elevated approach contrasts with the teaching of mathematics in a more mechanical way, as if students were machines, performing tasks without the need for thinking. We are developing the human being through all of our pedagogical endeavors, none the least, through math.

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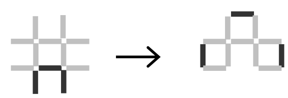
Solutions to puzzles:

1) See diagram at the right.



2) a) 15, 21, 28, 36, 45, 55, 66, 78, 91, 105

b) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

3) a) 30 $(=2^1 \cdot 3^1 \cdot 5^1)$, 24 $(=2^3 \cdot 3^1)$, 54 $(=2^1 \cdot 3^3)$, and 128 $(=2^7)$ are some of the possible answers.

b) 16 $(=2^4)$, 81 $(=3^4)$, and 625 $(=5^4)$ are some of the possible answers.

c) 4096 $(=2^{12})$ and 531,441 $(=3^{12})$ are two of the possible answers.

d) 3600

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[4] Rudolf Steiner, *Spiritual Ground of Education*, lecture given at Oxford, August 21 1922, GA 305