

3 Methods for Constructing a Projectivity

(Given 3 pairs of corresponding points)

Background:

The *Fundamental Theorem of Projective Geometry* states that

“A projectivity is fully determined by three points on one line, and three corresponding points on another line.”

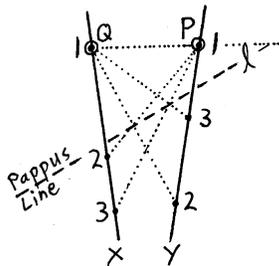
In other words, once you know three pairs of corresponding points, a fourth point on either line has a fixed place where its corresponding point on the other line must occur.

The Question:

Given points 1, 2, 3 on line X, and three corresponding points on line Y, how is it possible to construct a projectivity (i.e., determine proper placement of the intermediate line ℓ , and the points of perspectivity P and Q) such that the projectivity correctly projects the points from line X to line Y?

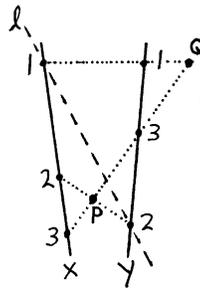
Solutions: I know of three possible methods to solve this problem.

Method #1



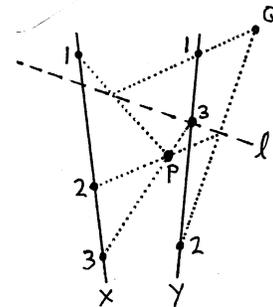
The Pappus line is used as line ℓ , and points P and Q coincide with a pair of corresponding points.

Method #2



Line ℓ is chosen so that it goes through two non-corresponding points. This then determines where P and Q must be placed.

Method #3



Choose one of the six points. Draw line ℓ through this chosen point. Also, place P or Q (you'll have to think about which one) somewhere on the line that connects the chosen point with its corresponding point. This determines where the other point of perspectivity (P or Q) must be.