

Introducing Vertical Subtraction (a.k.a. “Borrowing”) in Third Grade

Notes:

- There are many ways that we could do 832–485. Certainly one way is to line up the numbers vertically, and then follow the procedure known as “borrowing” (as shown on the right).
- Our goal in third grade is to build up to this process slowly so that the students understand why borrowing works.
- Here are some other ways to calculate 832–485:
 - First subtract 500 from 832 (to get 332), and then recognize that we just subtracted 15 too much – so we add 332+15 to get 347.
 - Subtract 32 from both numbers, and then do 800-453 to get 347.
 - Count up from 485 to 832, in stages. Going from 485 to 500 is 15. Going from 500 to 800 is 300. Going from 800 to 832 is 32. Adding together the underlined numbers gives us 347.
 - Regroup 832 to 700+120+12, and then subtract from that 400+80+5 to get 347. This is actually what is behind the borrowing method (shown above)!
- *Why not introduce carrying and borrowing in second grade?* Although the above methods for solving 832-485 may be beyond most second graders, second graders can develop a sense of number and higher-level mathematical thinking by seeing and experiencing different methods for solving such problems. If we introduce carrying and borrowing in second grade, students are less likely to understand the thinking behind the procedure and are more likely to rely exclusively on the procedure instead of “thinking through” simpler problems, like 53-47.

$$\begin{array}{r} 7\cancel{8}^{12}\cancel{3}^{12} \\ - 485 \\ \hline 347 \end{array}$$

Here is a possible progression for introducing vertical subtraction:

Step #0: Mental math prerequisites: practice problems like 16-9, 800-300, and 150-90.

Step #1a: Two-digit subtraction without borrowing.

$$\begin{array}{r} 56 \rightarrow 50+6 \\ -23 \rightarrow -(20+3) \\ \hline 33 \leftarrow 30+3 \end{array}$$

Step #1b: The same without all the writing.

$$\begin{array}{r} 56 \\ -23 \\ \hline 33 \end{array}$$

Step #2a: With regrouping/borrowing.

$$\begin{array}{r} 52 \rightarrow 50+2 \rightarrow 40+12 \\ -28 \rightarrow -(20+8) \rightarrow -(20+8) \\ \hline 24 \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow 20+4 \end{array}$$

Step #2b: The same without all the writing.

$$\begin{array}{r} \cancel{5}^{12} \\ -28 \\ \hline 24 \end{array}$$

Step #3a: Three-digits – no borrowing (for the second math block).

$$\begin{array}{r} 678 \rightarrow 600+70+8 \\ -435 \rightarrow -(400+30+5) \\ \hline 243 \leftarrow 200+40+3 \end{array}$$

Step #3b: The same without all the writing.

$$\begin{array}{r} 678 \\ -435 \\ \hline 243 \end{array}$$

Step #4a: Three-digits – regrouping/borrowing once.

$$\begin{array}{r} 983 \rightarrow 900+80+3 \rightarrow 900+70+13 \\ -326 \rightarrow -(300+20+6) \rightarrow -(300+20+6) \\ \hline 657 \leftarrow \leftarrow \leftarrow \leftarrow 600+50+7 \end{array}$$

Step #4b: The same without all the writing.

$$\begin{array}{r} 9\cancel{8}^{13} \\ -326 \\ \hline 657 \end{array}$$

Step #5a: Three-digits – regrouping/borrowing twice.

$$\begin{array}{r} 832 \rightarrow 800+30+2 \rightarrow 800+20+12 \rightarrow 700+120+12 \\ -485 \rightarrow -(400+80+5) \rightarrow -(400+80+5) \rightarrow -(400+80+5) \\ \hline 347 \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow 300+40+7 \end{array}$$

Step #5b: The same without all the writing.

$$\begin{array}{r} 7\cancel{8}^{12}\cancel{3}^{12} \\ -485 \\ \hline 347 \end{array}$$

Dealing with zeroes (like 803-687) could wait until fourth grade.